

# MATHEMATICS

## Chapter 11: Constructions



## Constructions

1. To divide a line segment internally in a given ratio  $m : n$ , where both  $m$  and  $n$  are positive integers, we follow the steps given below:

Step 1: Draw a line segment  $AB$  of given length by using a ruler.

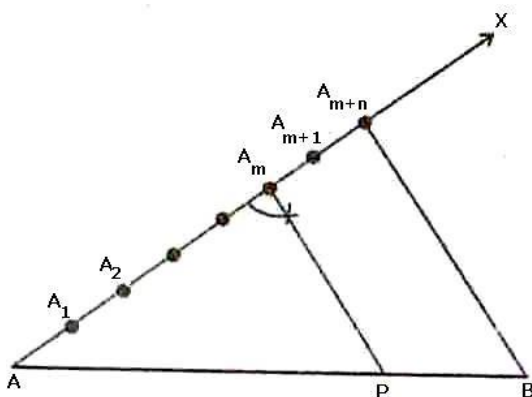
Step 2: Draw any ray  $AX$  making an acute angle with  $AB$ .

Step 3: Along  $AX$  mark off  $(m + n)$  points  $A_1, A_2, \dots, A_{m-1}, A_{m+1}, \dots, A_{m+n}$ , such that  $AA_1 = A_1A_2 = A_{m+n-1}A_{m+n}$ .

Step 4: Join  $BA_{m+n}$ .

Step 5: Through the point  $A_m$ , draw a line parallel to  $A_{m+n}B$  by making an angle equal to  $\angle AA_{m+n}B$  at  $A_m$ , intersecting  $AB$  at point  $P$ .

The point  $P$  so obtained is the required point which divides  $AB$  internally in the ratio  $m : n$ .



### Justification

In  $\triangle ABA_{m+n}$ , we observe that  $A_mP$  is parallel to  $A_{m+n}B$ . Therefore, by Basic Proportionality theorem, we have:

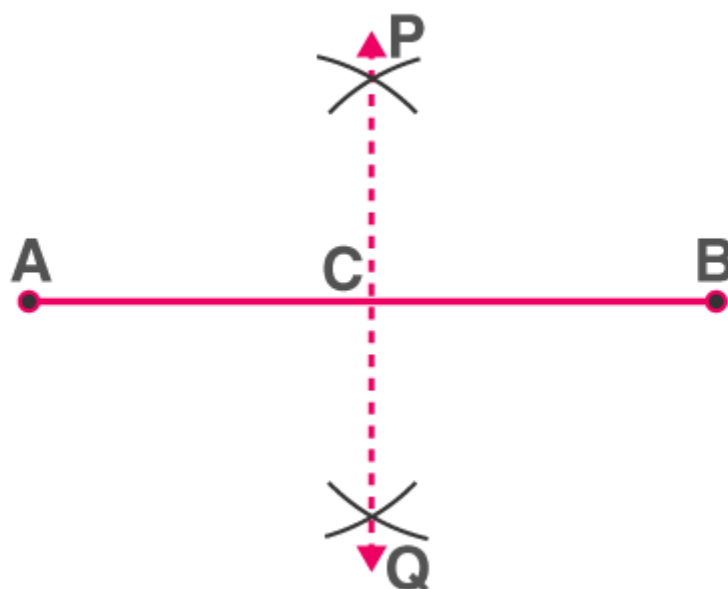
$$\begin{aligned} \frac{AA_m}{A_mA_{m+n}} &= \frac{AP}{PB} \\ &= \frac{AP}{PB} = \frac{m}{n} \left[ \because \frac{AA_m}{A_mA_{m+n}} = \frac{m}{n}, \text{ by construction} \right] \\ &= AP : PB = m : n \end{aligned}$$

Hence,  $P$  divides  $AB$  in the ratio  $m : n$ .

### Bisecting a Line Segment

**Step 1:** With a radius of more than half the length of the line segment, draw arcs centred at either end of the line segment so that they intersect on either side of the line segment.

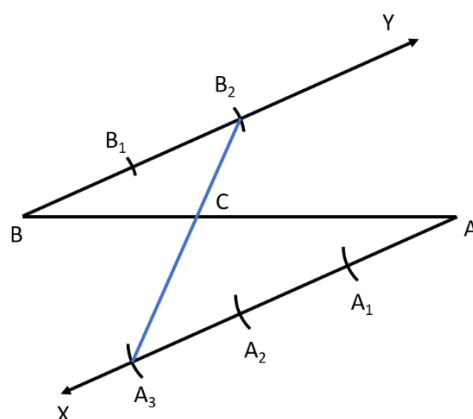
**Step 2:** Join the points of intersection. The line segment is bisected by the line segment joining the points of intersection.



PQ is the perpendicular bisector of AB

## 2. Alternative method to divide a line segment internally in a given ratio $m:n$

Example Find the point C such that it divides BA in ratio 2:3



### Steps of Construction:

- Draw any ray XA making an acute angle with BA.
- Draw a ray YB parallel to XA by making  $\angle YBA$  equal to  $\angle XAB$ .
- Locate the points  $A_1, A_2, A_3$  ( $m = 3$ ) on AX and  $B_1, B_2$  ( $n = 2$ ) on BY such that  $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2$ .
- Join  $A_3B_2$ . Let it intersect AB at a point C Then  $BC : CA = 2:3$

### Justification

Here  $\triangle BB_2C \sim \triangle AA_3C$  ... AA test

$$\frac{BB_2}{AA_3} = \frac{BC}{AC} \dots \dots (\text{c. p. s. t.})$$

$$\frac{2}{3} = \frac{BC}{AC}$$

3. The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as a **scale factor**. The scale factor may be less or greater than 1.
4. If the scale factor is less than 1, then the new figure will be smaller in comparison to the given figure.
5. If the scale factor is greater than 1, then the new figure will be bigger in comparison to the given figure.

### Construction of Triangle Similar to given Triangle

Consider a triangle ABC. Let us construct a triangle similar to  $\triangle ABC$  such that each of its sides is  $\frac{m}{n}$  of the corresponding sides of  $\triangle ABC$ .

**Steps of constructions when  $m < n$ :**

Step 1: Construct the given triangle ABC by using the given data.

Step 2: Take any one of the three side of the given triangle as base. Let AB be the base of the given triangle.

Step 3: At one end, say A, of base AB. Construct an acute angle  $\angle BAX$  below the base AB.

Step 4: Along AX mark off n points  $A_1, A_2, A_3, \dots, A_n$  such that

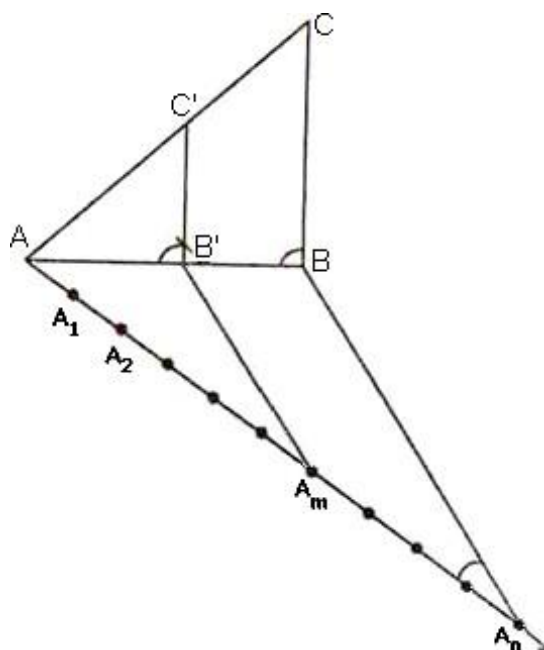
$$AA_1 = A_1A_2 = \dots = A_{n-1}A_n$$

Step 5: Join  $A_nB$

Step 6: Draw  $A_mB'$  parallel to  $A_nB$  which meets AB at  $B'$ .

Step 7: From  $B'$  draw  $B'C' \parallel BC$  meeting AC at  $C'$ .

Triangle  $AB'C'$  is the required triangle each of whose sides is  $\frac{m}{n}$  of the corresponding side of  $\triangle ABC$ .



**Justification**

Since  $A_m B' \parallel A_n B$ . Therefore

$$\frac{AB'}{B'B} = \frac{AA_m}{A_m A_n} \text{ [by basic proportionality theorem]}$$

$$\Rightarrow \frac{AB'}{B'B} = \frac{m}{n-m}$$

$$\Rightarrow \frac{B'B}{AB'} = \frac{n-m}{m}$$

$$\text{Now, } \frac{AB}{AB'} = \frac{AB' + B'B}{AB'}$$

$$\Rightarrow \frac{AB}{AB'} = 1 + \frac{B'B}{AB'} = 1 + \frac{n-m}{m} + \frac{n}{m}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{m}{n}$$

In triangles  $ABC$  and  $AB'C'$ , we have

$$\angle BAC = \angle B'AC'$$

$$\text{And } \angle ABC = \angle AB'C'$$

So, by AA similarity criterion, we have

$$\Delta AB'C' \sim \Delta ABC$$

$$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{m}{n}$$

### Steps of construction when $m > n$ :

Step 1: Construct the given triangle by using the given data.

Step 2: Take any one of the three sides of the given triangle and consider it as the base. Let AB be the base of the given triangle.

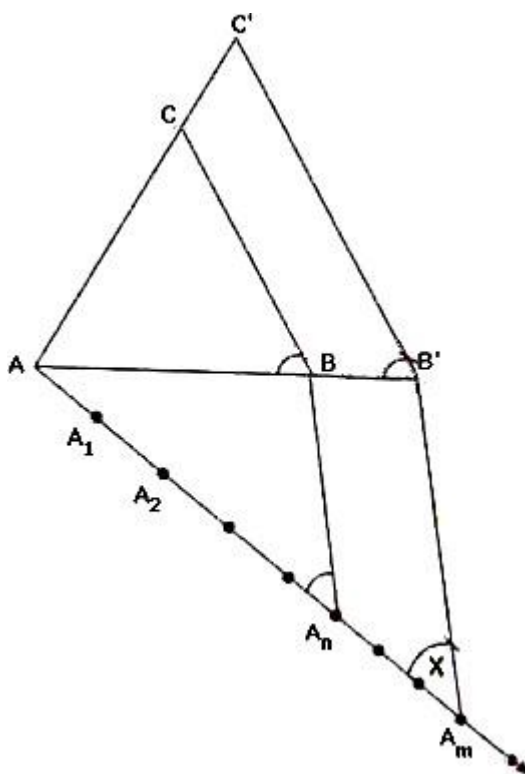
Step 3: At one end, say A, of base AB. Construct an acute angle  $\angle BAX$  below base AB i.e., on the opposite side of the vertex C.

Step 4: Along AX mark off m (large of m and n) points  $A_1, A_2, A_3, \dots, A_m$  of AX such that

$$AA_1 = A_1A_2 = \dots = A_{m-1}A_m.$$

Step 5: Join  $A_nB$  to B and draw a line through  $A_m$  parallel to  $A_nB$ , intersecting the extended line segment AB at  $B'$ .

Step 6: Draw a line through  $B'$  parallel to BC intersecting the extended line segment AC at  $C'$ . Step 7:  $\triangle AB'C'$  so obtained is the required triangle.



### Justification

Consider triangle ABC and  $AB'C'$ . We have:

$$\angle BAC = \angle B'AC'$$

$$\angle ABC = \angle AB'C'$$

So, by AA similarity criterion,

$$\triangle ABC \sim \triangle AB'C'$$

$$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'}$$

$$\Delta A A_m B', A_n B \parallel A_m B'$$

$$\therefore \frac{AB}{BB'} = \frac{AA_n}{A_n A_m}$$

$$\Rightarrow \frac{BB'}{AB} = \frac{A_n A_m}{AA_n}$$

$$\Rightarrow \frac{BB'}{AB} = \frac{m-n}{n}$$

$$\Rightarrow \frac{AB' - AB}{AB} = \frac{m-n}{n}$$

$$\Rightarrow \frac{AB'}{AB} - 1 = \frac{m-n}{n}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{m}{n}$$

From (i) and (ii), we have

$$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{m}{n}$$

The tangent to a circle is a line that intersects the circle at exactly one point.

Tangent to a circle is perpendicular to the radius through the point of contact.

### Construction of Triangle to a Circle from a point outside the Circle

**Construction of a tangent to a circle from a point outside the circle, when its centre is known**

The steps of constructions are as follows:

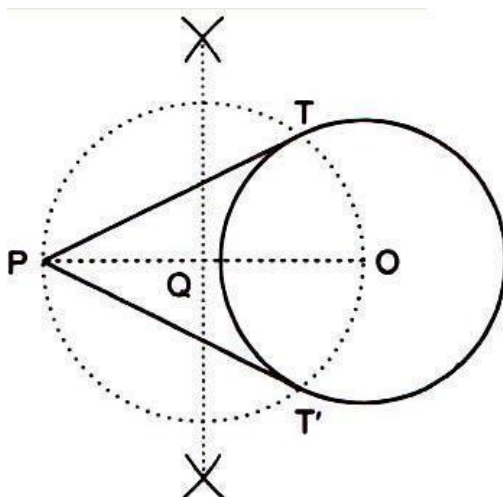
Step 1: Join the centre O of the circle to the point P.

Step 2: Draw perpendicular bisector of OP intersecting OP at Q.

Step 3: With Q as centre and radius OQ, draw a circle. This circle has OP as its diameter.

Step 4: Let this circle intersect the first circle at two points T and T'. Join PT and P T' .

PT and P T' are the two tangents to the given circle from the point P.



### Justification

Join OT and OT'

It can be seen that  $\angle PTO$  is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

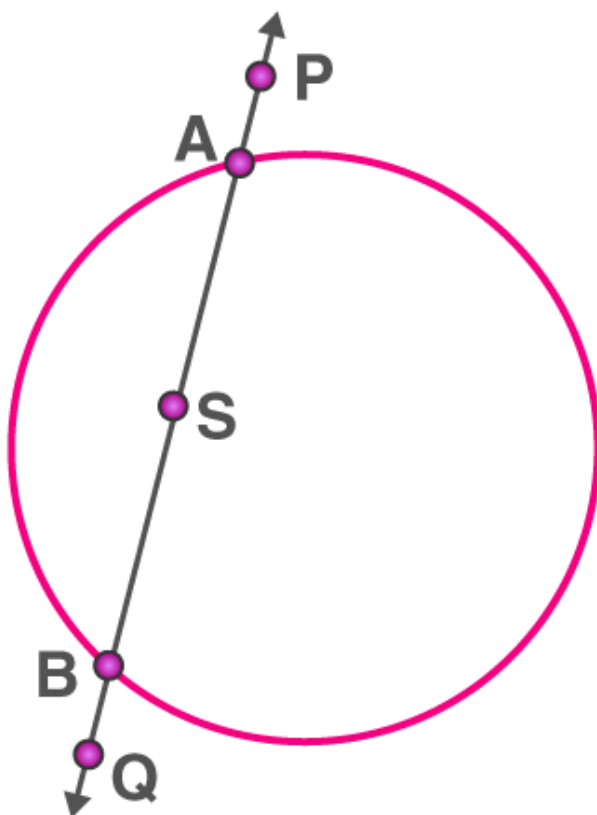
$$\therefore \angle PTO = 90^\circ$$

$$\Rightarrow OT \perp PT$$

Since OT is the radius of the circle, PT has to be a tangent of the circle. Similarly, PT' is a tangent of the circle.

### Number of Tangents to a circle from a given point

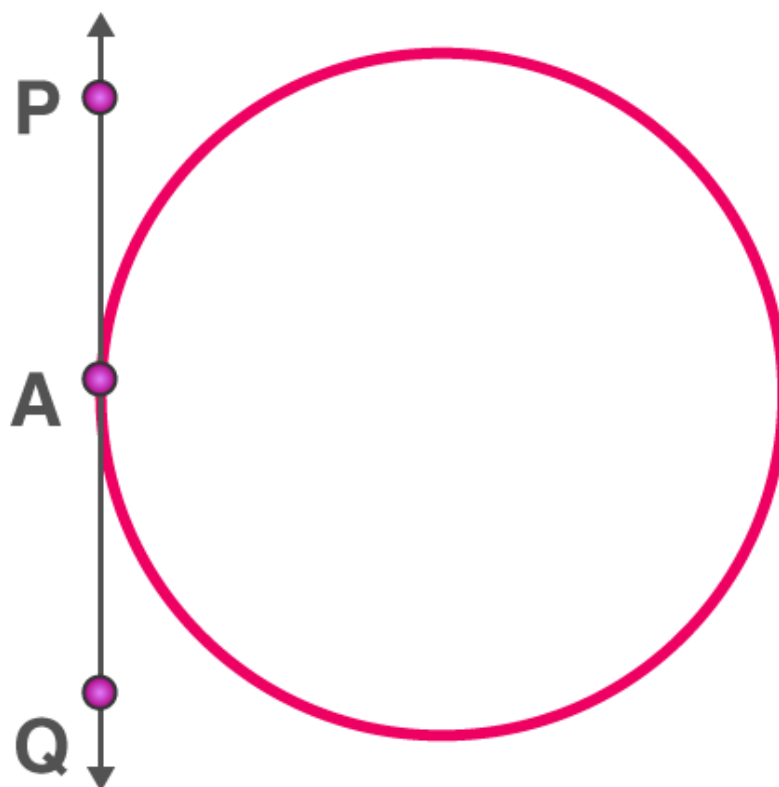
If the point is in an interior region of the circle, any line through that point will be a secant. So, in this case, there is no tangent to the circle.



AB is a secant drawn through the point S

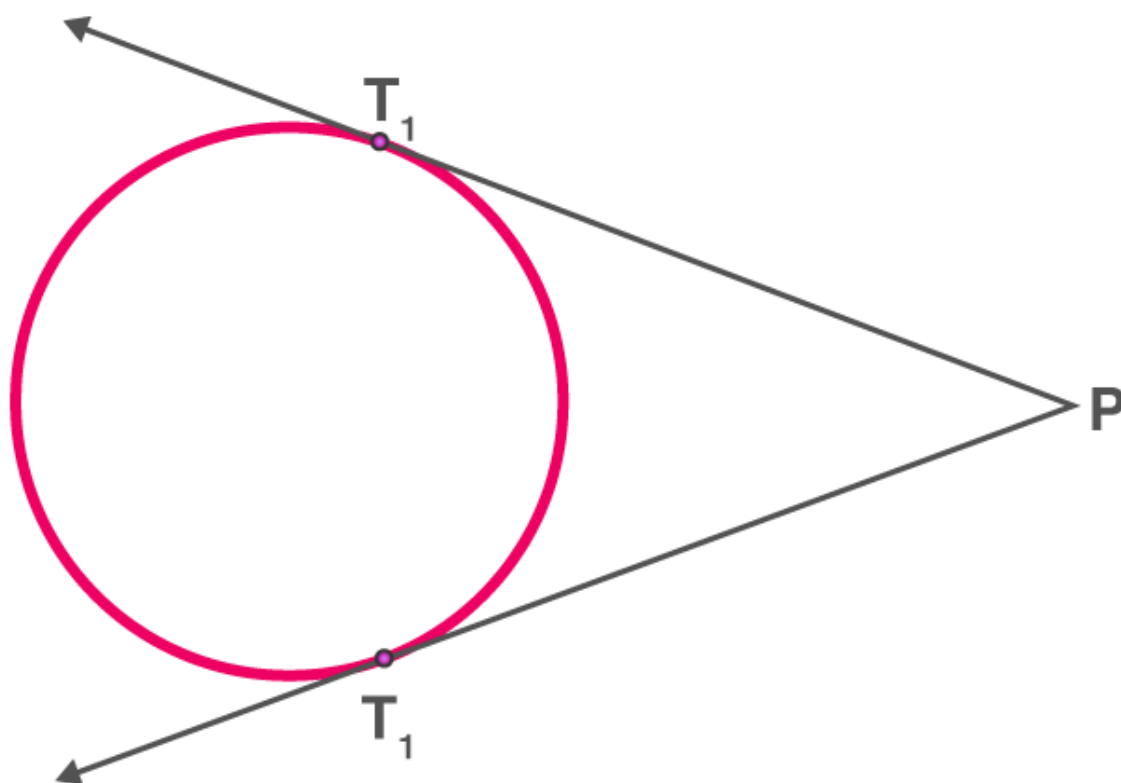
When the point lies on the circle, there is accurately only one tangent to a circle.





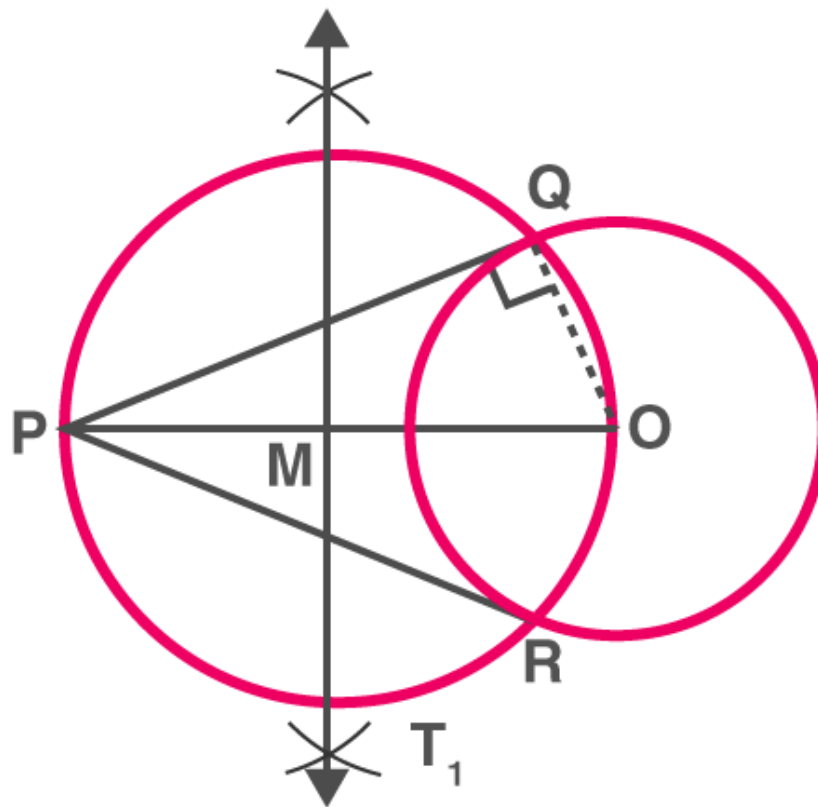
PQ is the tangent touching the circle at A

When the point lies outside of the circle, there are exactly two tangents to a circle.



PT<sub>1</sub> and PT<sub>2</sub> are tangents touching the circle at T<sub>1</sub> and T<sub>2</sub>

**Drawing tangents to a circle from a point outside the circle**



To construct the tangents to a circle from a point outside it.

Consider a circle with centre  $O$  and let  $P$  be the exterior point from which the tangents to be drawn.

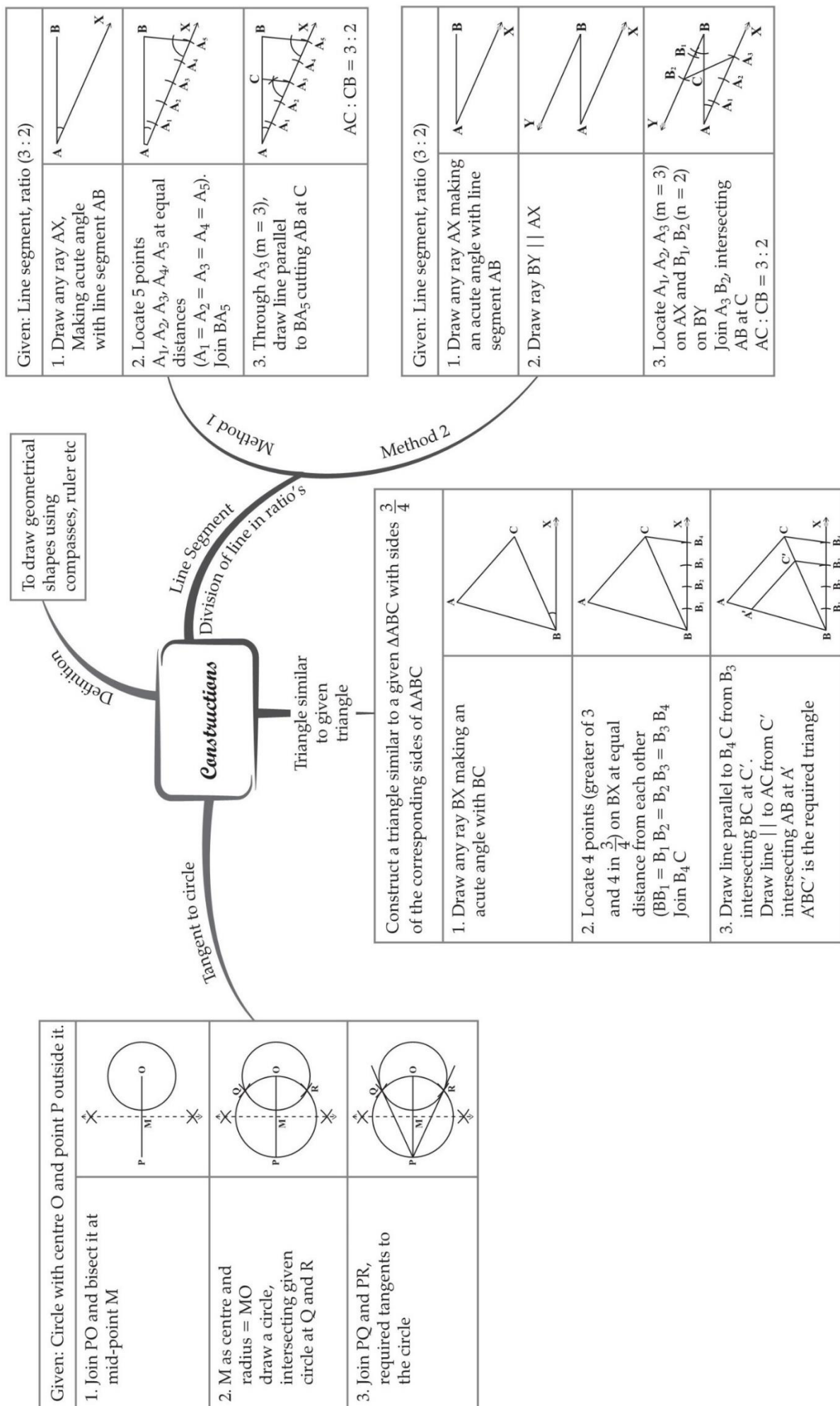
**Step 1:** Join the  $PO$  and bisect it. Let  $M$  be the midpoint of  $PO$ .

**Step 2:** Taking  $M$  as the centre and  $MO$  (or  $MP$ ) as radius, draw a circle. Let it intersect the given circle at the points  $Q$  and  $R$ .

**Step 3:** Join  $PQ$  and  $PR$

**Step 4:**  $PQ$  and  $PR$  are the required tangents to the circle.

# MIND MAP : LEARNING MADE SIMPLE



## Important Questions

### Multiple Choice questions-

1. To divide a line segment AB in the ratio  $p : q$  ( $p, q$  are positive integers), draw a ray AX so that  $\angle BAX$  is an acute angle and then mark points on ray AX at equal distances such that the minimum number of these points is

(a) greater of  $p$  and  $q$

(b)  $p + q$

(c)  $p + q - 1$

(d)  $pq$

2. To draw a pair of tangents to a circle which are inclined to each other at an angle of  $35^\circ$ . It is required to draw tangents at the end points of those two radii of the circle, the angle between which is

(a)  $105^\circ$

(b)  $70^\circ$

(c)  $140^\circ$

(d)  $145^\circ$

3. To divide a line segment AB in the ratio  $5 : 7$ , first a ray AX is drawn so that  $\angle BAX$  is an acute angle and then at equal distances points are marked on the ray AX such that the minimum number of these points is

(a) 8

(b) 10

(c) 11

(d) 12

4. To divide a line segment AB in the ratio  $4 : 7$ , ray AX is drawn first such that  $\angle BAX$  is an acute angle and then points  $A_1, A_2, A_3, \dots$  are located at equal distances on the ray AX and the point B is joined to

(a)  $A_{12}$

(b)  $A_{11}$

(c)  $A_{10}$

(d)  $A_9$

5. To divide a line segment AB in the ratio 5 : 6, draw a ray AX such that  $\angle BAX$  is an acute angle, then draw a ray B4 parallel to AX and the points  $A_1, A_2, A_3, \dots$  and  $B_1, B_2, B_3, \dots$  are located at equal distances on ray AX and B4, respectively. Then the points joined are:

- (a)  $A_5$  and  $B_6$
- (b)  $A_6$  and  $B_5$
- (c)  $A_4$  and  $B_5$
- (d)  $A_5$  and  $B_4$

6. To construct a triangle similar to a given  $\triangle ABC$  with its sides  $\frac{3}{7}$  of the corresponding sides of  $\triangle ABC$ , first draw a ray BX such that  $\angle CBX$  is an acute angle and X lies on the opposite side of A with respect to BC. Then locate points  $B_1, B_2, B_3, \dots$  on BX at equal distances and next step is to join

- (a)  $B_{10}$  to C
- (b)  $B_3$  to C
- (c)  $B_7$  to C
- (d)  $B_4$  to C

7. To construct a triangle similar to a given  $\triangle ABC$  with its sides  $\frac{8}{5}$  of the corresponding sides of  $\triangle ABC$  draw a ray BX such that  $\angle CBX$  is an acute angle and X is on the opposite side of A with respect to BC. Then minimum number of points to be located at equal distances on ray BX is

- (a) 5
- (b) 8
- (c) 13
- (d) 3

8. To draw a pair of tangents to a circle which are inclined to each other at an angle of  $60^\circ$ , it is required to draw tangents at end points of those two radii of the circle, the angle between them should be:

- (a)  $135^\circ$
- (b)  $90^\circ$
- (c)  $60^\circ$

(d)  $120^\circ$

9. To construct a pair of tangents to a circle at an angle of  $60^\circ$  to each other, it is needed to draw tangents at endpoints of those two radii of the circle, the angle between them should be:

(a)  $100^\circ$

(b)  $90^\circ$

(c)  $180^\circ$

(d)  $120^\circ$

10. A pair of tangents can be constructed from a point P to a circle of radius 3.5 cm situated at a distance of \_\_\_\_\_ from the centre.

(a) 3.5 cm

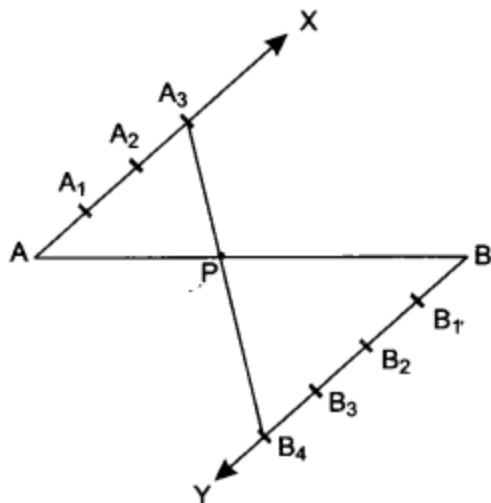
(b) 2.5 cm

(c) 5 cm

(d) 2 cm

### Very Short Questions:

1. Is construction of a triangle with sides 8 cm, 4 cm, 4 cm possible?
2. To divide the line segment AB in the ratio 5 : 6, draw a ray AX such that  $\angle BAX$  is an acute angle, then draw a ray BY parallel to AX and the point  $A_1, A_2, A_3, \dots$  and  $B_1, B_2, B_3, \dots$  are located at equal distances on ray AX and BY respectively. Then which points should be joined?
3. To draw a pair of tangents to a circle which are inclined to each other at an angle of  $60^\circ$ , it is required to draw tangents at end points of those two radii of the circle. What should be the angle between them?
4. In Fig. by what ratio does P divide AB internally.



5. Given a triangle with side  $AB = 8$  cm. To get a line segment  $AB' = 2$  of  $AB$ , in what ratio will line segment  $AB$  be divided?
6. Draw a line segment of length 6 cm. Using compasses and ruler, find a point  $P$  on it which divides it in the ratio 3 : 4.
7. Draw a line segment  $AB$  of length 7 cm. Using ruler and compasses, find a point  $P$  on  $AB$  such that  $\frac{AP}{AB} = \frac{3}{5}$ .

### Short Questions :

1. Draw a triangle  $ABC$  in which  $AB = 5$  cm,  $BC = 6$  cm and  $\angle ABC = 60^\circ$ . Then construct a triangle whose sides are 57 times the corresponding sides of  $\triangle ABC$ .
2. Construct a triangle with sides 5 cm, 5.5 cm and 6.5 cm. Now construct another triangle, whose sides are  $\frac{3}{5}$  times the corresponding sides of the given triangle.
3. Construct a right triangle in which the sides, (other than the hypotenuse) are of length 6 cm and 8 cm. Then construct another triangle, whose sides are  $\frac{3}{5}$  times the corresponding sides of the given triangle.
4. Draw a triangle  $PQR$  such that  $PQ = 5$  cm,  $\angle P = 120^\circ$  and  $PR = 6$  cm. Construct another triangle whose sides are  $\frac{3}{4}$  times the corresponding sides of  $\triangle PQR$ .
5. Draw a triangle  $ABC$  with  $BC = 7$  cm,  $\angle B = 45^\circ$  and  $\angle C = 60^\circ$ . Then construct another triangle, whose sides are  $\frac{3}{5}$  times the corresponding sides of  $\triangle ABC$ .
6. Construct a triangle with sides 5 cm, 4 cm and 6 cm. Then construct another triangle whose sides are  $\frac{2}{3}$  times the corresponding sides of first triangle.
7. Draw a pair of tangents to a circle of radius 3 cm, which are inclined to each other at an angle of  $60^\circ$ .

8. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm.
9. Draw a pair of tangents to a circle of radius 4.5 cm, which are inclined to each other at an angle of  $45^\circ$ .
10. Draw two tangents to a circle of radius 3.5 cm, from a point P at a distance of 6.2 cm from its centre.

### Long Questions :

1. Draw a triangle ABC with side  $BC = 7$  cm,  $\angle B = 45^\circ$  and  $\angle A = 105^\circ$ . Then construct a triangle whose sides are  $\frac{3}{5}$  times the corresponding sides of  $\triangle ABC$ .
2. Draw a triangle ABC with side  $BC = 6$  cm,  $\angle C = 30^\circ$  and  $\angle A = 105^\circ$ . Then construct another triangle whose sides are  $\frac{2}{3}$  times the corresponding sides of  $\triangle ABC$ .
3. Draw a triangle with sides 5 cm, 6 cm and 7 cm. Then construct another triangle whose sides are  $\frac{2}{3}$  times the corresponding sides of the first triangle.
4. Construct an isosceles triangle whose base is 6 cm and altitude 4 cm. Then construct another triangle whose sides are  $\frac{3}{4}$  times the corresponding sides of the isosceles triangle.
5. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.
6. Construct a  $\triangle ABC$  in which  $AB = 6$  cm,  $\angle A = 30^\circ$  and  $\angle B = 60^\circ$ . Construct another  $\triangle AB'C'$  similar to  $\triangle ABC$  with base  $AB' = 8$  cm.
7. Construct a triangle ABC in which  $AB = 5$  cm,  $BC = 6$  cm and  $\angle ABC = 60^\circ$ . Now construct another triangle whose sides are  $\frac{5}{7}$  times the corresponding sides of  $\triangle ABC$ .
8. Construct a triangle ABC in which  $BC = 6$  cm,  $AB = 5$  cm and  $\angle ABC = 60^\circ$ . Then construct another triangle whose sides are  $\frac{3}{4}$  times the corresponding sides of  $\triangle ABC$ .

### Case Study Questions:

1. The management of a school decided to arouse interest of their students in Mathematics. So they want to construct some geometrical shapes in one corner of the school premises. They showed a rough sketch of a right triangular structure on a plane sheet of paper with sides  $AB = 6$  m,  $BC = 8$  m and  $\angle B = 90^\circ$ . The diagram shows a



- i. Referring to the above, what is the length of perpendicular drawn on side AC from vertex B?
  - a. 2.6 m
  - b. 3.0 m
  - c. 4.8 m
  - d. 4.0 m
- ii. Referring to the above, what is the length of perpendicular drawn on side AC from vertex B?
  - a. 2.6 m
  - b. 3.0 m
  - c. 4.8 m
  - d. 4.0 m
- iii. Referring to the above, the length of tangent AE is
  - a. 10 m
  - b. 8 m
  - c. 12 m
  - d. 6 m

iv. Referring to the above, what will be the length of AD?

- a. 3.6 m
- b. 3.8 m
- c. 4.8 m
- d. 5.6 m

v. Referring to the above, sum of angles  $\angle BAE$  and  $\angle BOE$  is

- a.  $120^\circ$
- b.  $180^\circ$
- c.  $90^\circ$
- d.  $60^\circ$

2. The construction of a road is in progress. A road already exists through a forest that goes over a circular lake. The engineer wants to build another road through the forest that connects this road but does not go through the lake.

As it turns out, the road the engineer will be building and the road it will connect to both represent characteristics of a circle that have their own name. The road/bridge that already exists is called a secant of the circular lake, and the road the engineer is going to build is called the tangent of the circular lake.

i. Refer to the question (2) if the road under construction, PT is 6 km and it is inclined at an angle of  $30^\circ$  to the line joining the centre, the radius of the lake is

- a.  $3\sqrt{3}$  km
- b.  $4\sqrt{3}$  km
- c.  $2\sqrt{3}$  km
- d.  $5\sqrt{3}$  km

ii. Refer to the above, if PT = 12 km and PA = 9 km, then the length of existing bridge is

- a. 7 km
- b. 9 km
- c. 12 km
- d. 16 km

- iii. Refer to the question (3) above, the area of the lake is
- $12\pi \text{ km}^2$
  - $16\pi \text{ km}^2$
  - $18\pi \text{ km}^2$
  - $9\pi \text{ km}^2$
- iv. Refer to the above if the length of existing bridge is 5 km and the length of the existing road outside the lake is 4 km, then the length of the road under construction is
- 4 km
  - 6 km
  - 10 km
  - 14 km
- v. Refer to the question (3) above, the circumference of the lake is
- $2\sqrt{3} \pi \text{ km}$
  - $3\sqrt{3} \pi \text{ km}$
  - $4\sqrt{3} \pi \text{ km}$
  - $5\sqrt{3} \pi \text{ km}$

### Assertion Reason Questions-

**1. Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- Both A and R are true and R is the correct explanation of A.
- Both A and R are true and R is not the correct explanation of A.
- A is true but R is false.
- Both A and R is false.

**Assertion:** a, b and c are the lengths of three sides of a triangle, then  $a+b > c$

**Reason:** The sum of two sides of a triangle is always greater than the third side.

**2. Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- Both A and R are true and R is the correct explanation of A.
- Both A and R are true and R is not the correct explanation of A.

- (c) A is true but R is false.
- (d) Both A and R is false.

**Assertion:** The side lengths 4cm, 4cm and 4cm can be sides of equilateral triangle.

**Reason:** Equilateral triangle has all its three sides equal.

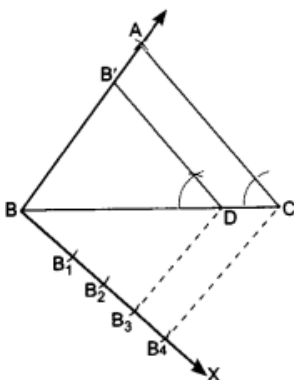
## Answer Key-

### Multiple Choice questions-

1. (b)  $p + q$
2. (d)  $145^\circ$
3. (d) 12
4. (b)  $A_{11}$
5. (a)  $A_5$  and  $B_6$
6. (c)  $B_7$  to C
7. (b) 8
8. (d)  $120^\circ$
9. (d)  $120^\circ$
10. (c) 5 cm

### Very Short Answer :

1. No, we know that in a triangle sum of two sides of a triangle is greater than the third side. So the condition is not satisfied.
2.  $A_5$  and  $B_6$ .
3.  $120^\circ$
4. From Fig. it is clear that there are 3 points at equal distances on AX and 4 points at equal distances on BY. Here P divides AB on joining  $A_3 B_4$ . So P divides internally by 3 : 4.
- 5.



Given  $AB = 8$  cm

$$AB' = \frac{3}{4} \text{ of } AB$$

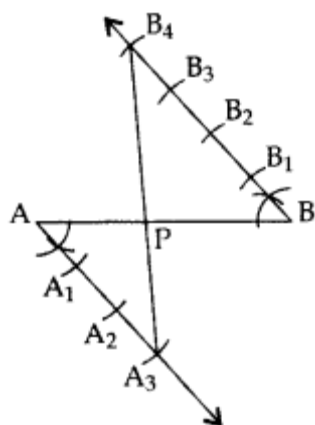
$$= \frac{3}{4} \times 8 = 6 \text{ cm}$$

$$BB' = AB - AB' = 8 - 6 = 2 \text{ cm.}$$

$$\Rightarrow AB' : BB' = 6 : 2 = 3 : 1$$

Hence the required ratio is 3 : 1.

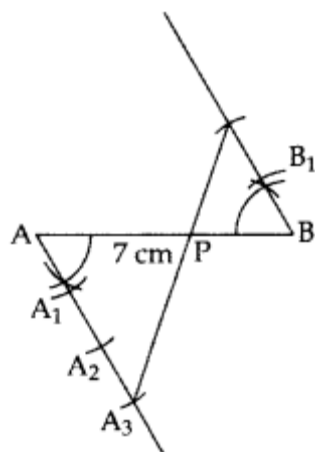
6.



Hence,  $PA : PB = 3 : 4$

7.  $AB = 7 \text{ cm}$ ,  $AB = \frac{AP}{AB} = \frac{3}{5} \dots$  [Given

$$\therefore AP : PB = 3 : 2$$



Hence,  $AP : AB = 3 : 5$  or  $\frac{AP}{AB} = \frac{3}{5}$

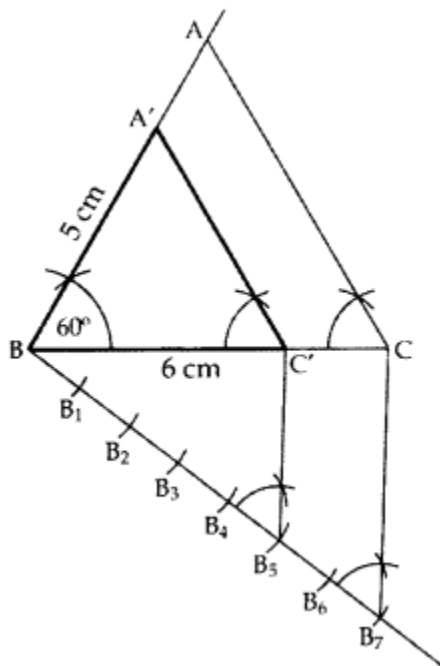
## Short Answer :

1. In  $\triangle ABC$

$$AB = 5 \text{ cm}$$

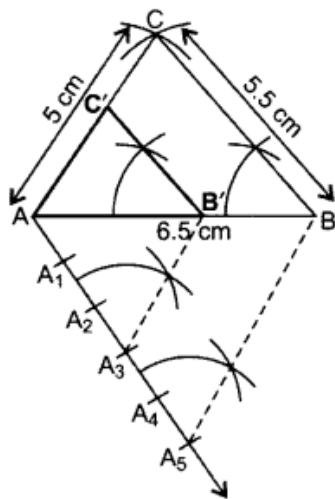
$BC = 6 \text{ cm}$

$\angle ABC = 60^\circ$



Hence,  $\Delta A'BC'$  is the required  $\Delta$ .

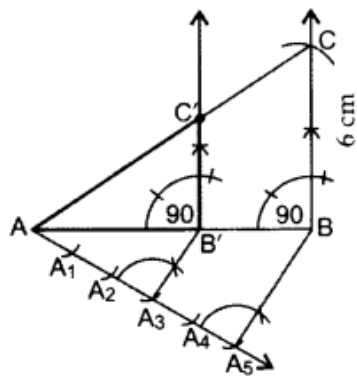
2.



$\therefore \Delta AB'C'$  is the required  $\Delta$ .

3. Here  $AB = 8 \text{ cm}$ ,  $BC = 6 \text{ cm}$  and

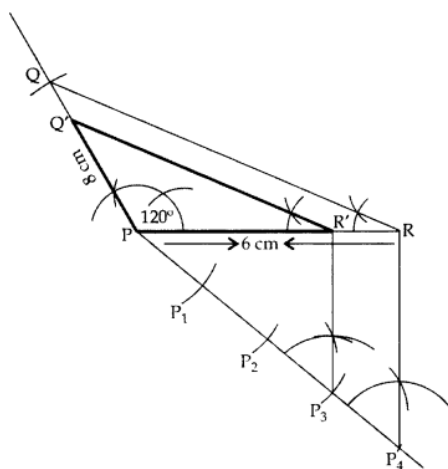
Ratio  $= \frac{3}{5}$  of corresponding sides



$\therefore \triangle AB'C'$  is the required triangle.

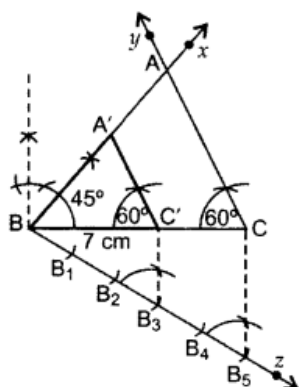
4. In  $\triangle PQR$ ,

$PQ = 5 \text{ cm}$ ,  $PR = 6 \text{ cm}$ ,  $\angle P = 120^\circ$



$\therefore \triangle PQR'$  is the required  $\triangle$ .

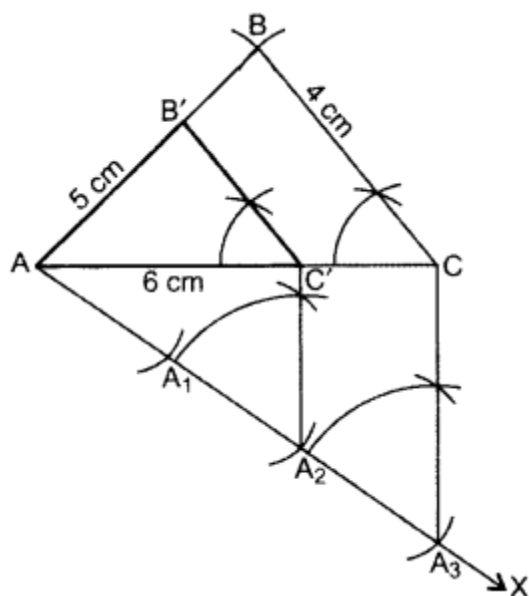
5. Here,  $BC = 7 \text{ cm}$ ,  $\angle B = 45^\circ$ ,  $\angle C = 60^\circ$  and ratio is  $\frac{3}{5}$  times of corresponding sides



$\therefore \triangle A'B'C'$  is the required triangle.

6.





### Steps of Construction:

Draw  $\triangle ABC$  with  $AC = 6$  cm,  $AB = 5$  cm,  $BC = 4$  cm.

Draw ray  $AX$  making an acute angle with  $AC$ .

Locate 3 equal points  $A_1, A_2, A_3$  on  $AX$ .

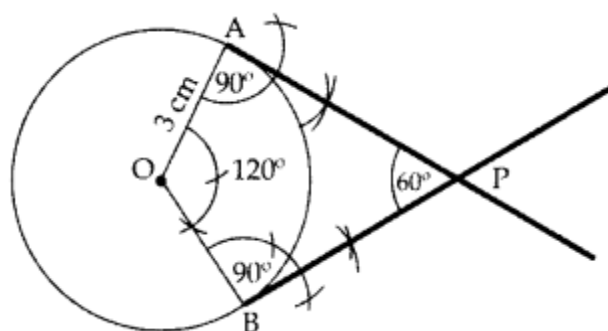
Join  $CA_3$ .

Join  $A_2C' \parallel CA_3$ .

From point  $C'$  draw  $B'C' \parallel BC$ .

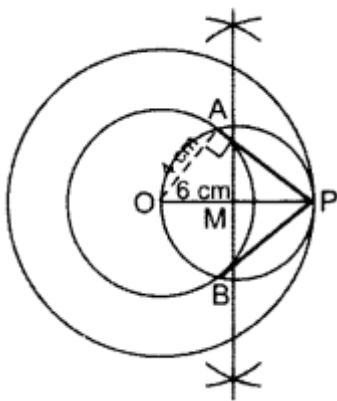
$\therefore \triangle AB'C'$  is the required triangle.

7.



$\therefore PA$  &  $PB$  are the required tangents. +

8.



### Steps of Construction:

Draw two circles with radius  $OA = 4$  cm and  $OP = 6$  cm with  $O$  as centre. Draw  $\perp$  bisector of  $OP$  at  $M$ . Taking  $M$  as centre and  $OM$  as radius draw another circle intersecting the smaller circle at  $A$  and  $B$  and touching the bigger circle at  $P$ . Join  $PA$  and  $PB$ .  $PA$  and  $PB$  are the required tangents.

Verification:

In rt.  $\triangle OAP$ ,

$$OA^2 + AP^2 = OP^2 \dots [\text{Pythagoras' theorem}]$$

$$(4)^2 + (AP)^2 = (6)^2$$

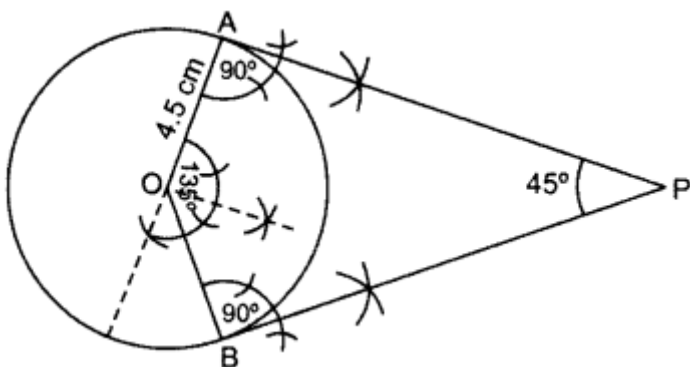
$$AP^2 = 36 - 16 = 20$$

$$AP = +\sqrt{20} = \sqrt{4 \times 5}$$

$$= 2\sqrt{5} \quad 2(2.236) = 4.472 = 4.5 \text{ cm}$$

By measurement,  $\therefore PA = PB = 4.5$  cm

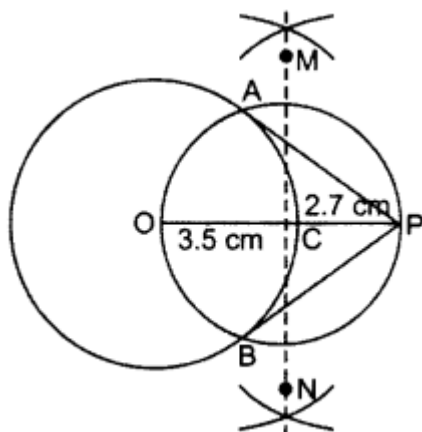
9.



Draw  $\angle AOB = 135^\circ$ ,  $\angle OAP = 90^\circ$ ,  $\angle OBP = 90^\circ$

$\therefore PA$  and  $PB$  are the required tangents.

10.  $OP = OC + CP = 3.5 + 2.7 = 6.2$  cm



Hence AP & PB are the required tangents.

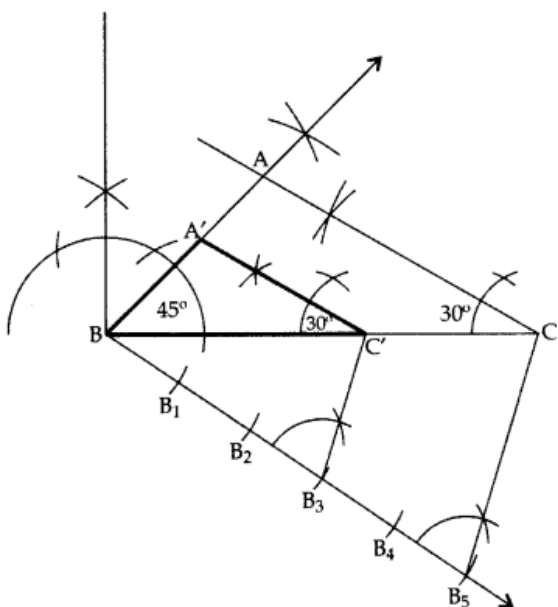
### Long Answer :

1. In  $\triangle ABC$ ,  $\angle A + \angle B + \angle C = 180^\circ$  ... [angle sum property of a  $\triangle$

$$105^\circ + 45^\circ + C = 180^\circ$$

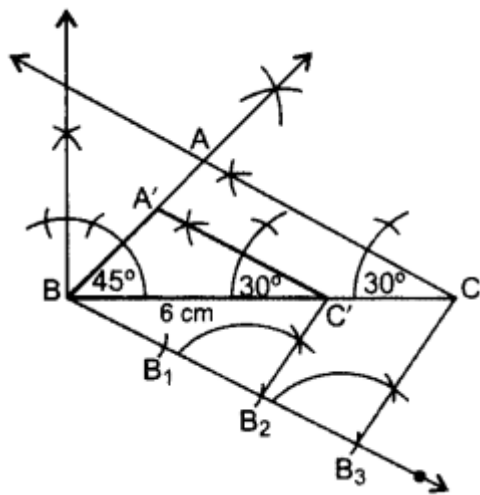
$$\angle C = 180^\circ - 105^\circ - 45^\circ = 30^\circ$$

$$BC = 7 \text{ cm}$$



$\therefore \triangle A'BC'$  is the required  $\triangle$ .

2. Here,  $BC = 6 \text{ cm}$ ,  $\angle A = 105^\circ$  and  $\angle C = 30^\circ$



In  $\triangle ABC$ ,

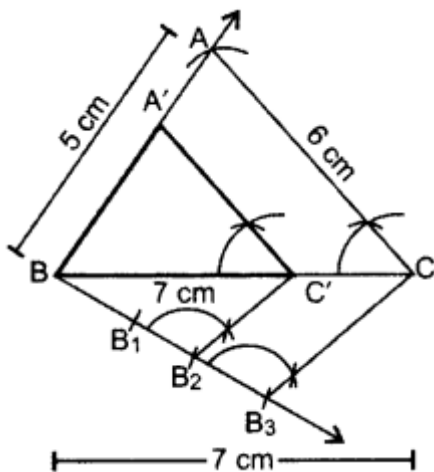
$\angle A + \angle B + \angle C = 180^\circ$  ...[Angle-sum-property of a  $\Delta$

$$105^\circ + \angle B + 30^\circ = 180^\circ$$

$$\angle B = 180^\circ - 105^\circ - 30^\circ = 45^\circ$$

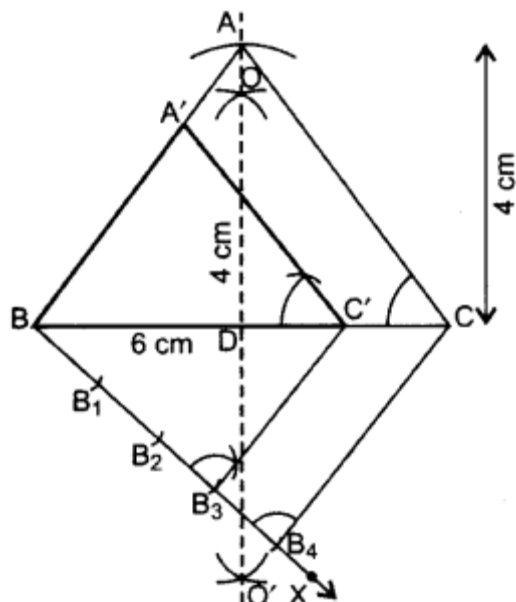
$\therefore \triangle A'BC'$  is the required  $\Delta$ .

3. Here,  $AB = 5$  cm,  $BC = 7$  cm,  $AC = 6$  cm and ratio is  $\frac{2}{3}$  times of corresponding sides.



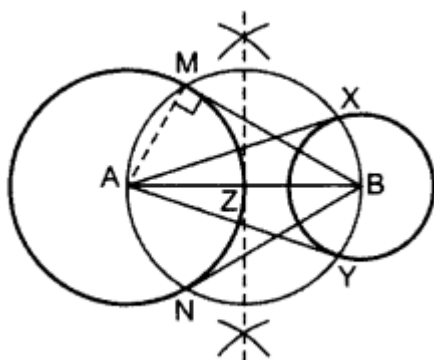
$\therefore \triangle A'BC'$  is the required triangle.

4.



$\therefore \triangle A'BC'$  is the required triangle.

5.



### Step of constructions:

Draw two circles on A and B as asked.

Z is the mid-point of AB.

From Z, draw a circle taking  $ZA = ZB$  as radius,

so that the circle intersects the bigger circle at M and N and smaller circle at X and Y.

Join AX and AY, BM and BN.

BM, BN are the required tangents from external point B.

AX, AY are the required tangents from external point A.

Justification:

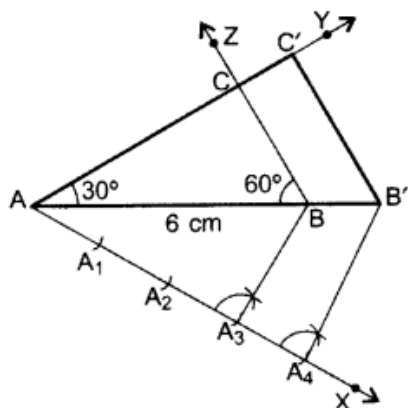
$\angle AMB = 90^\circ$  ...[Angle in a semi-circle

Since, AM is a radius of the given circle.

$\therefore$  BM is a tangent to the circle

Similarly, BN, AX and AY are also tangents.

6.

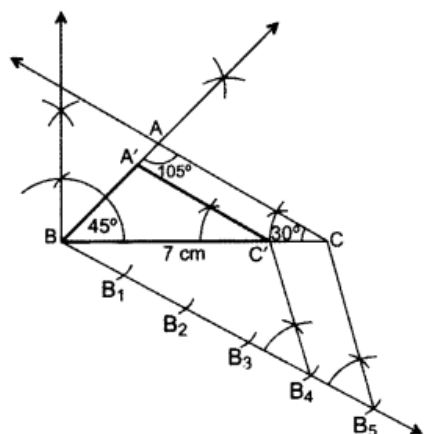


**Steps of construction:**

- Draw a  $\triangle ABC$  with side  $AB = 6$  cm,  $\angle A = 30^\circ$  and  $\angle B = 60^\circ$ .
- Draw a ray  $AX$  making an acute angle with  $AB$  on the opposite side of point  $C$ .
- Locate points  $A_1, A_2, A_3$  and  $A_4$  on  $AX$ .
- Join  $A_3B$ . Draw a line through  $A_4$  parallel to  $A_3B$  intersecting extended  $AB$  at  $B'$ .
- Draw a line parallel to  $BC$  intersecting ray  $AY$  at  $C'$ .

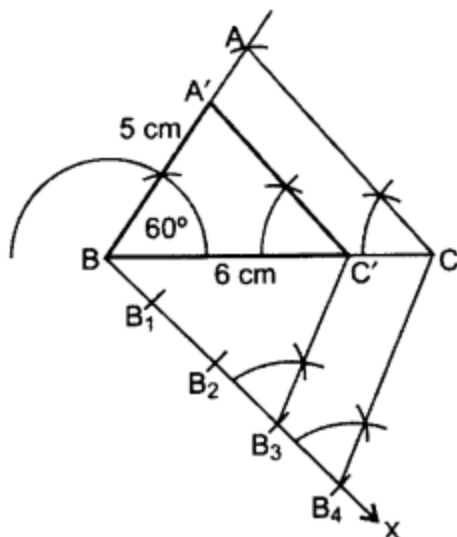
Hence,  $\triangle AB'C'$  is the required triangle.

7. In  $\triangle ABC$ ,  $AB = 5$  cm;  $BC = 6$  cm;  $\angle ABC = 60^\circ$



$\therefore \triangle A'BC'$  is the required  $\triangle$ .

8.



### Steps of Construction:

- Draw  $\triangle ABC$  with the given data.
- Draw a ray  $BX$  downwards making an acute angle with  $BC$ .
- Locate 4 points  $B_1, B_2, B_3, B_4$ , on  $BX$ , such that  $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$ .
- Join  $CB_4$ .
- From  $B_3$  draw a line  $C'B_3 \parallel CB_4$  intersecting  $BC$  at  $C'$ .
- From  $C'$  draw  $A'C' \parallel AC$  intersecting  $AB$  at  $A'$ .

Then  $\triangle A'B'C'$  is the required triangle.

Justification:

$$\triangle A'BC' \sim \triangle ABC \quad \dots[\text{AA similarity rule}]$$

$$\frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} \quad \dots(i)$$

$\dots[\text{Sides are proportional}]$

$$\text{But } \frac{BC'}{BC} = \frac{BB_3}{BB_4} = \frac{3}{4} \quad \dots(ii)$$

$$\therefore \frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{3}{4} \quad \dots[\text{From (i) \& (ii)}]$$

### Case Study Answer:

1. Answer:

- c.
- d.
- d.
- a.
- b.

2. Answer:

- i. c.
- ii. a.
- iii. a.
- iv. b.
- v. c.

Assertion Reason Answer-

- (a) Both A and R are true and R is the correct explanation of A.
- (a) Both A and R are true and R is the correct explanation of A.