MATHEMATICS

Chapter 7: Coordinate Geometry

Coordinate Geometry

1. **Coordinate axes:**

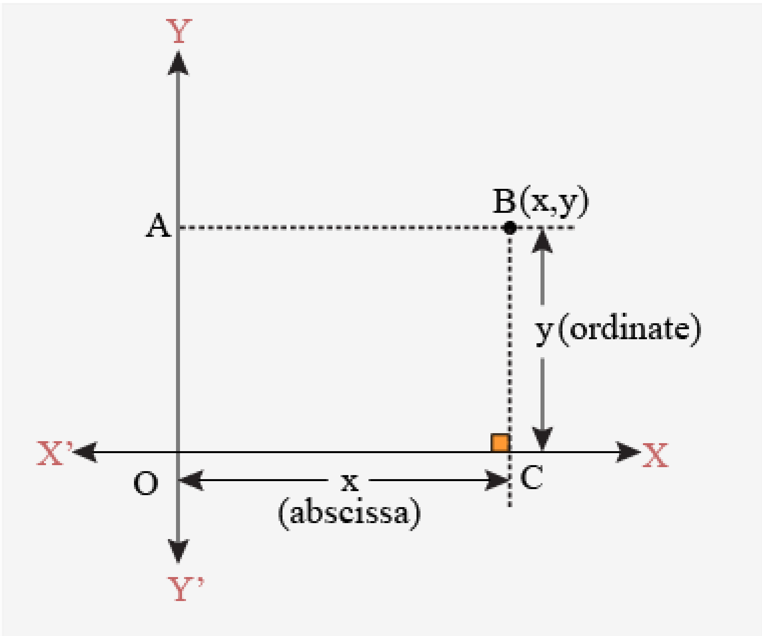
   Two perpendicular number lines intersecting at point zero are called **coordinate axes**. The point of intersection is called **origin** and denoted by '*O*'. The horizontal number line is the ***x*-axis** (denoted by *X'OX*) and the vertical one is the ***y*-axis** (denoted by *Y'OY*).

2. **Cartesian plane** is a plane formed by the coordinate axes perpendicular to each other in the plane. It is also called as *xy* plane.

   The axes divide the Cartesian plane into four parts called the **quadrants** (one fourth part), numbered I, II, III and IV anticlockwise from *OX*.

   Points on a Cartesian Plane

   A pair of numbers locate points on a plane called the coordinates. The distance of a point from the y-axis is known as abscissa or x-coordinate. The distance of a point from the x-axis is called ordinates or y-coordinate.



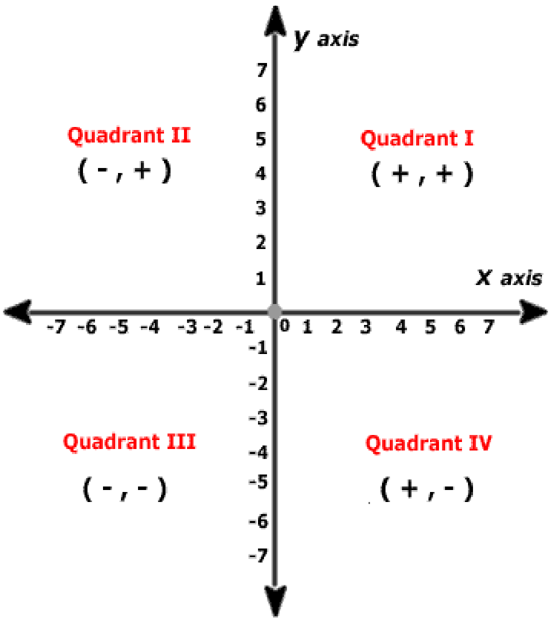
Representation of (x,y) on the cartesian plane

3. **Coordinates of a point:**
   - The x-coordinate of a point is its perpendicular distance from *y*-axis, called **abscissa.**
   - The y-coordinate of a point is its perpendicular distance from *x*-axis, called **ordinate**
   - If the abscissa of a point is *x* and the ordinate of the point is *y*, then (*x, y*) is called the **coordinates** of the point.
   - The point where the *x*-axis and the *y*-axis intersect is represented by the coordinate point (0, 0) and is called the **origin**.

4. **Sign of the coordinates in the quadrants:**

   Sign of coordinates depicts the quadrant in which it lies.
   - The point having both the coordinates positive i.e. of the form (+, +) will lie in the first quadrant.

- The point having x-coordinate negative and y-coordinate positive i.e. of the form (-, +) will lie in the second quadrant.
- The point having both the coordinates negative i.e. of the form (-, -) will lie in the third quadrant.
- The point having x-coordinate positive and y-coordinate negative i.e. of the form (+,-) will lie in the fourth quadrant.



5. Coordinates of a point on the x-axis or y-axis:

The coordinates of a point lying on the *x*-axis are of the form (*x*, 0) and that of the point on the *y*-axis are of the form (0, *y*).

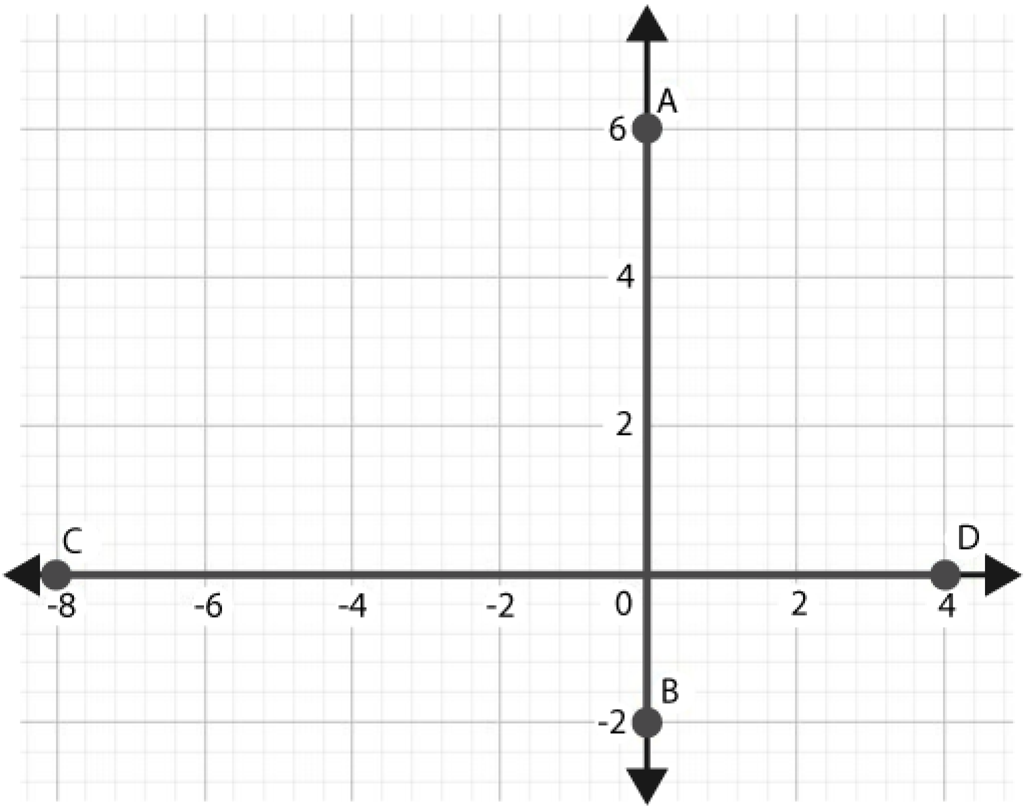
6. Distance formula

The distance formula is used to find the distance between two any points say $P(x_1, y_1)$ and $Q(x_2, y_2)$ which is given by: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- The distance of a point P(x, y) from the origin O(0, 0) is $OP\sqrt{x^2 + y^2}$
- The points *A, B* and *C* are **collinear** if *AB + BC = AC*.

Distance between Two Points on the Same Coordinate Axes

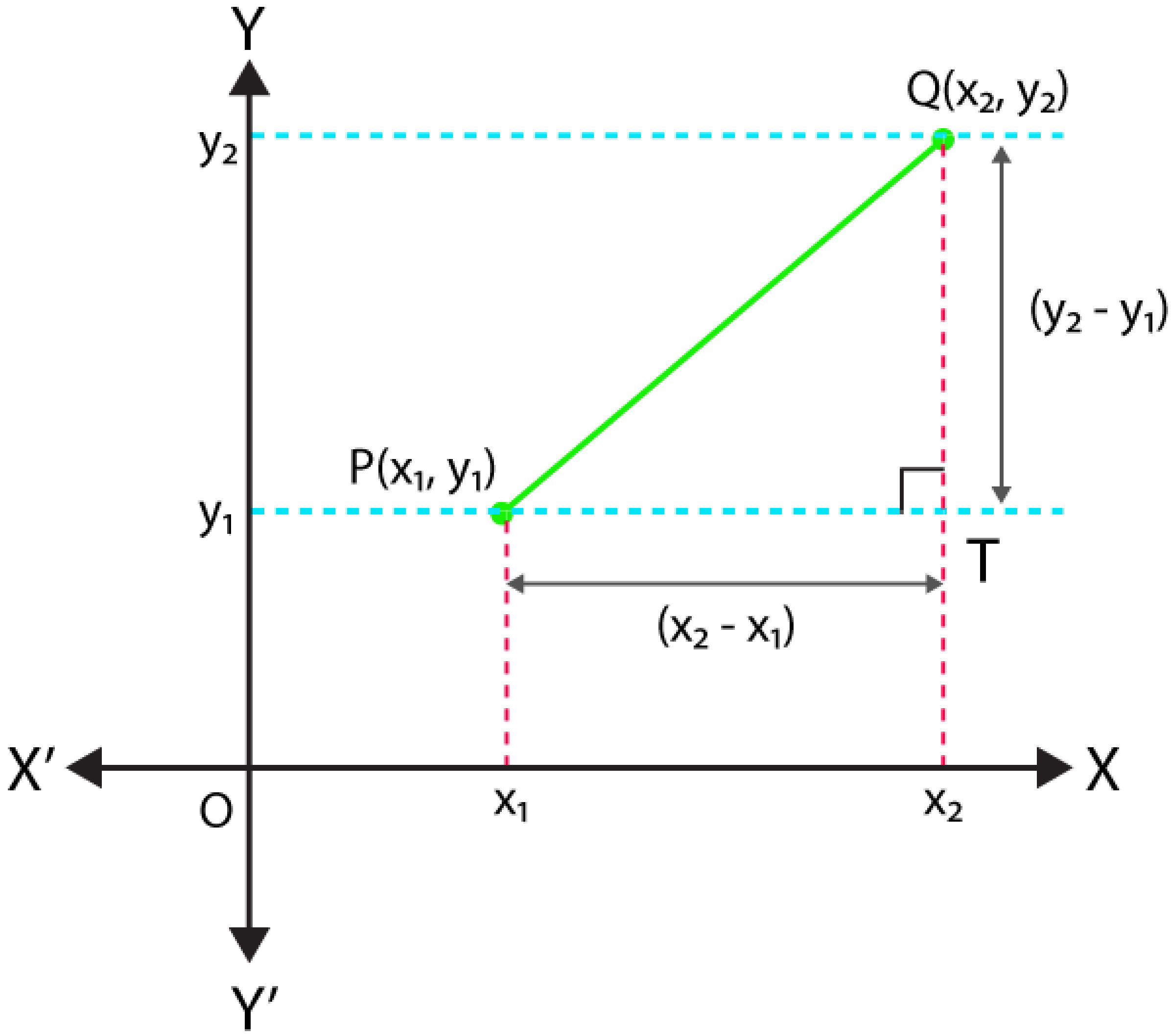
The distance between two points that are on the same axis (x-axis or y-axis), is given by the difference between their ordinates if they are on the y-axis, else by the difference between their abscissa if they are on the x-axis.



Distance AB = 6 – (-2) = 8 units

Distance CD = 4 – (-8) = 12 units

Distance between Two Points Using Pythagoras Theorem



Finding distance between 2 points using Pythagoras Theorem

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two points on the cartesian plane.

Draw lines parallel to the axes through P and Q to meet at T.

ΔPTQ is right-angled at T.

By Pythagoras Theorem,

$PQ^2 = PT^2 + QT^2$

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$

$PQ = \sqrt{[x_2 - x_1)^2 + (y_2 - y_1)^2]}$

7. Determining the type of triangle using distance formula

i. Three points *A, B* and *C* are the vertices of an **equilateral triangle** if *AB = BC = CA*.

ii. The points *A, B* and *C* are the vertices of an **isosceles triangle** if *AB = BC or BC = CA or CA = AB*.

iii. Three points *A, B* and *C* are the vertices of *a* **right triangle** if the sum of the squares of any two sides is equal to the square of the third side.

8. Determining the type of quadrilateral using distance formula

For the given four points A, B, C and D, if:

i. $AB = CD$, $BC = DA$; $AC \neq BD \Rightarrow ABCD$ is a parallelogram.

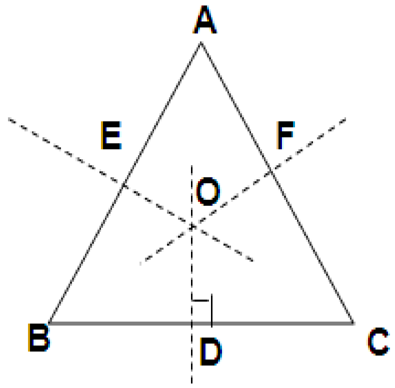
ii. $AB = BC = CD = DA$; $AC \neq BD \Rightarrow ABCD$ is a rhombus

iii. $AB = CD$, $BC = DA$; $AC = BD \Rightarrow ABCD$ is a rectangle

iv. $AB = BC = CD = DA$; $AC = BD \Rightarrow ABCD$ is a square.

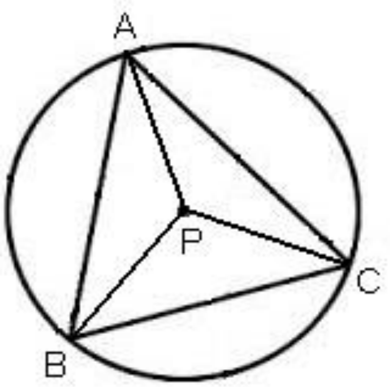
9. Circumcentre of a triangle

The point of intersection of the perpendicular bisectors of the sides of a triangle is called the **circumcentre**. In the figure, O is the circumcentre of the triangle ABC.



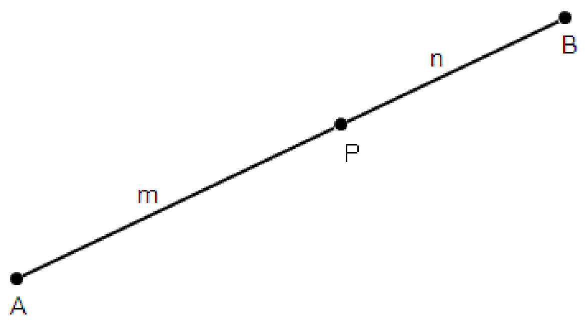
Circumcentre of a triangle is equidistant from the vertices of the triangle. That is, ***P* is the circumcentre of Δ *ABC*, if *PA* = *PB* = *PC***.

- Moreover, if a circle is drawn with *P* as centre and PA or PB or PC as radius, the circle will pass through all the three vertices of the triangle. *PA* (or *PB* or *PC*) is said to be the **circumradius** of the triangle.



10. Section formula

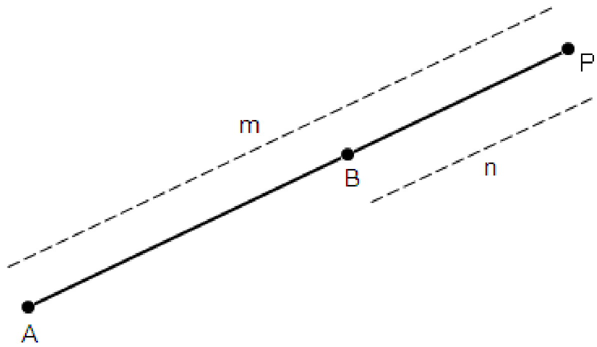
If *P* is a point lying on the line segment joining the points *A* and *B* such that $AP : BP = m : n$. Then, we say that the **point *P* divides the line segment *AB* internally** in the ratio $m : n$.



Coordinates of a point which divides the line segment joining the points (x_1, y_1) and $(x_2,$

y_2) in the ratio m: n internally are given by: $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+my_1}{m+n}\right)$ This is known as the **section formula**.

11. If P is a point lying on AB produced such that $AP: BP = m: n$, then point ***P* divides *AB* externally** in the ratio $m: n$.



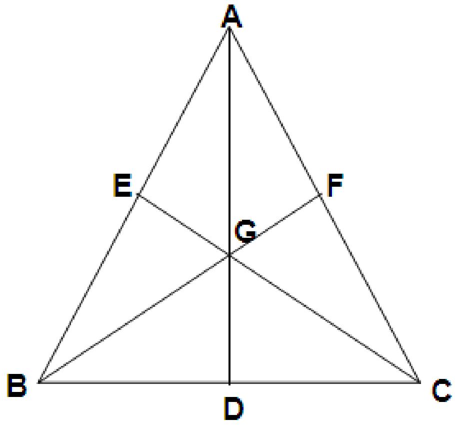
If P divides the line segment joining the points A (x_1, y_1) and B (x_2, y_2) in the ratio m: n externally, then the coordinates of point P are given by $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-my_1}{m-n}\right)$

12. **Coordinates of Mid-point**

Mid-point divides the line segment in the ratio 1:1. Coordinates of the mid-point of a line segment joining the points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

13. **Centroid of a triangle**

The point of intersection of the three medians of a triangle is called the centroid.



In the figure, G is the centroid of the triangle ABC where AD, BF and CE are the medians through A, B and C respectively.

Centroid divides the median in the ratio of 2:1.

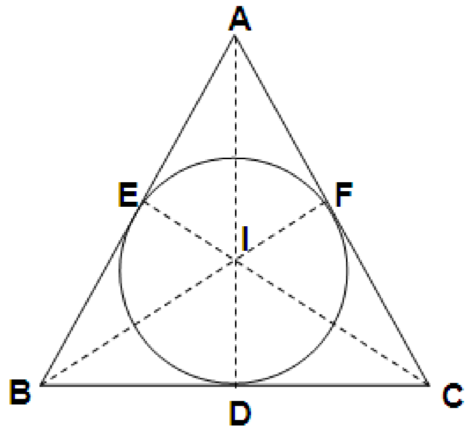
14. Coordinates of the centroid

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle ABC, then the **coordinates of the centroid** are given by $\mathrm{G}(\mathrm{x}, \mathrm{y}) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

15. Incentre of a triangle

The point of intersection of all the internal bisectors of the angles of a triangle is called the **incentre**.

It is also the centre of a circle which touches all the sides of a triangle (such type of a circle is named as the incircle).



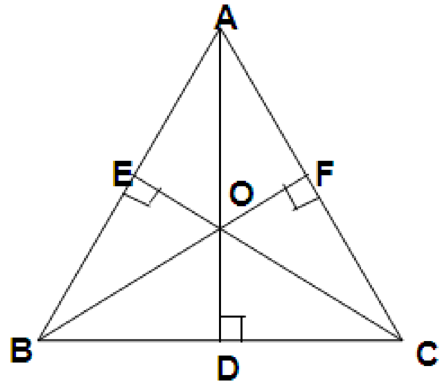
In the figure, *I* is the incentre of the triangle ABC.

16. Coordinates of incentre

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle, then the **coordinates of incentre** are given by $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

17. Orthocentre of a triangle

The point of intersection of all the perpendiculars drawn from the vertices on the opposite sides (called altitudes) of a triangle is called the **Orthocentre** which can be obtained by solving the equations of any two of the altitudes.



In the figure, O is the orthocentre of the triangle ABC.

18. If the triangle is equilateral, the centroid, the incentre, the orthocenter and the circumcentre coincides.

19. Orthocentre, centroid and circumcentre are always collinear, whereas the centroid divides the line joining the orthocentre and the circumcentre in the ratio of 2:1.

20. Area of a triangle

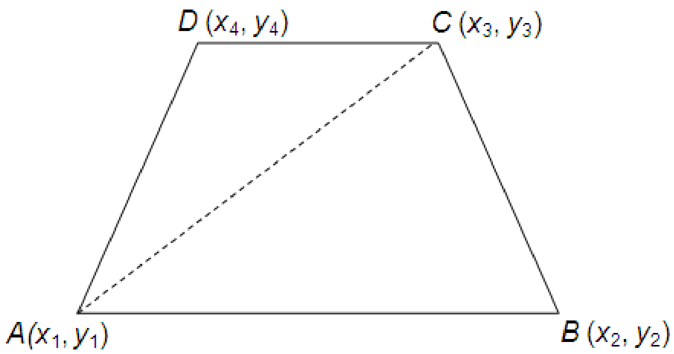
If A(x1, y1), B(x2, y2) and C(x3, y3) are the vertices of a triangle, then the area of triangle ABC is given by $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

- Three given points are **collinear**, if the **area of triangle formed by these points is zero.**

21. Area of a quadrilateral

Area of a quadrilateral can be calculated by dividing it into two triangles.

Area of quadrilateral ABCD = Area of ΔABC+ Area of ΔACD

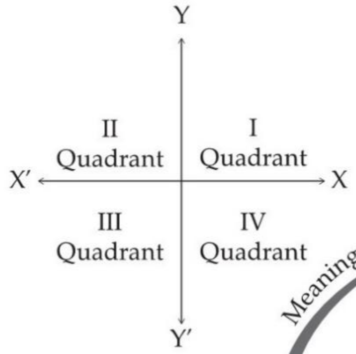


Note: To find the area of a polygon, divide it into triangular regions having no common area, then add the areas of these regions.

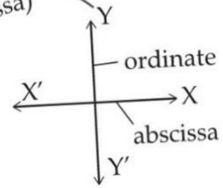
MIND MAP : LEARNING MADE SIMPLE

Coordinate Geometry

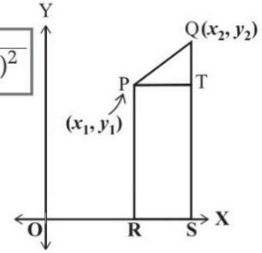
Meaning
Study of algebraic equations on graphs



Coordinate axis
- Horizontal: x–axis (Abscissa)
- Vertical: y–axis Ordinate



Distance formula
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example
Are the following points vertices of a square : (1, 7), (4, 2), (–1, –1), (–4, 4)?

A (1, 7); B = (4, 2); C = (–1, –1); D = (–4, 4)

$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{34}$

$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{34}$

$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{34}$

$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{34}$

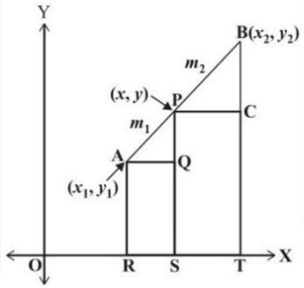
$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{68}$

$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{68}$

Since, AB = BC = CD = DA and AC = BD.
All four sides and diagonals are equal
Hence, ABCD is a square

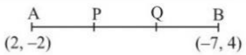
Section formula
Internally (+)
Externally (–)

$$\frac{m_1x_2 \pm m_2x_1}{m_1 \pm m_2}, \quad \frac{m_1y_2 \pm m_2y_1}{m_1 \pm m_2}$$



Example
Find point of Trisection of line segment AB, A(2, –2) and B(–7, 4)

Coordinate of P



$$= \left(\frac{1(-7) + 2(2)}{1+2}, \frac{1(4) + 2(-2)}{1+2}\right)$$

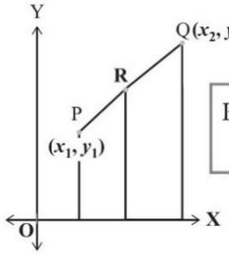
i.e., (–1, 0)

Coordinate of Q

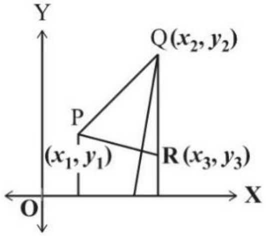
$$\left(\frac{2(-7) + 1(2)}{2+1}, \frac{2(4) + 1(-2)}{2+1}\right)$$

i.e., (–4, 2)

Mid-point Line Segment
$$R\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Area of Triangle
Area = $\frac{1}{2}\,|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]|$



Important Questions

Multiple Choice questions-

1. The ratio in which (4,5) divides the line segment joining the points (2,3) and (7,8) is

(a) 2:3

(b) -3:2

(c) 3:2

(d) -2:3

2. The values of x and y, if the distance of the point (x, y) from (-3,0) as well as from (3,0) is 4 are

(a) x = 1, y = 7

(b) x = 2, y = 7

(c) x = 0, y = – √7

(d) x = 0, y = ± √7

3.The distance between the points (3,4) and (8,-6) is

(a) 2√5 units

(b) 3√5 units

(c) √5 units

(d) 5√5 units

4. The ratio in which the x-axis divides the segment joining A(3,6) and B(12,-3) is

(a) 1:2

(b) -2:1

(c) 2:1

(d) -1:-1

5. The horizontal and vertical lines drawn to determine the position of a point in a Cartesian plane are called

(a) Intersecting lines

(b) Transversals

(c) Perpendicular lines

(d) X-axis and Y-axis

6. The mid point of the line segment joining A(2a,4) and B(-2,3b) is M (1,2a + 1). The values of a and b are

(a) 2,3

(b) 1,1

(c) -2,-2

(d) 2,2

7. The points (1,1), (-2, 7) and (3, -3) are

(a) vertices of an equilateral triangle

(b) collinear

(c) vertices of an isosceles triangle

(d) none of these

8. The line $3x + y - 9 = 0$ divides the line joining the points (1, 3) and (2, 7) internally in the ratio

(a) 3 : 4

(b) 3 : 2

(c) 2 : 3

(d) 4 : 3

9. The ordinate of a point is twice its abscissa. If its distance from the point (4,3) is $\sqrt{10}$, then the coordinates of the point are

(a) (1,2) or (3,6)

(b) (1,2) or (3,5)

(c) (2,1) or (3,6)

(d) (2,1) or (6,3)

10. The mid-point of the line segment joining the points A (-2, 8) and B (-6, -4) is

(a) (-4, -6)

(b) (2, 6)

(c) (-4, 2)

(d) (4, 2)

Very Short Questions:

1. What is the area of the triangle formed by the points 0 (0, 0), A (-3, 0) and B (5, 0)?

2. If the centroid of triangle formed by points P (a, b), Q (b, c) and R (c, a) is at the origin, what is the value of a + b + c?

3. AOBC is a rectangle whose three vertices are A (0, 3), 0 (0, 0) and B (5, 0). Find the length of its diagonal.

4. Find the value of a, so that the point (3, a) lie on the line 2x – 3y = 5.

5. Find distance between the points (0, 5) and (-5, 0).

6. Find the distance of the point (-6,8) from the origin.

7. If the distance between the points (4, k) and (1, 0) is 5, then what can be the possible values of k?

8. If the points A (1, 2), B (0, 0) and C (a, b) are collinear, then what is the relation between a and b?

9. Find the ratio in which the line segment joining the points (-3, 10) and (6, – 8) is divided by (-1, 6).

10. The coordinates of the points P and Q are respectively (4, -3) and (-1, 7). Find the abscissa of a point R on the line segment PQ such that $\frac{PR}{PQ} = \frac{3}{5}$.

Short Questions :

1. Write the coordinates of a point on x-axis which is equidistant from the points (-3, 4) and (2, 5).

2. Find the values of x for which the distance between the points P (2, -3) and Q (x, 5) is 10.

3. What is the distance between the points (10 cos 30°, 0) and (0, 10 cos 60°)?

4. In Fig. 6.8, if A(-1, 3), B(1, -1) and C (5, 1) are the vertices of a triangle ABC, what is the length of the median through vertex A?

5. Find the ratio in which the line segment joining the points P (3, -6) and Q (5,3) is divided by the x-axis.

6. Point P (5, -3) is one of the two points of trisection of the line segment joining the points A (7, -2) and B (1, -5). State true or false and justify your answer.

7. Show that ΔABC, where A(-2, 0), B(2, 0), C(0, 2) and APQR where P(-4, 0), Q(4, 0), R(0,4) are similar triangles.

OR

Show that ΔABC with vertices A(-2, 0), B(0, 2) and C(2, 0) is similar to ΔDEF with vertices D(-4, 0), F(4,0) and E(0, 4).

[ΔPQR is replaced by ΔDEF]

8. Point P (0, 2) is the point of intersection of y-axis and perpendicular bisector of line segment joining the points, A (-1, 1) and B (3, 3). State true or false and justify your answer.

9. Determine, if the points (1, 5), (2, 3) and (-2, -11) are collinear.

10. Find the distance between the following pairs of points:

(i) (-5, 7), (-1, 3)

(ii) (a, b), (-a, -b)

Long Questions :

1. Find the value of 'k", for which the points are collinear: (7, -2), (5, 1), (3, k).

2. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

3. Find the area of the quadrilateral whose vertices, taken in order, are (-4, -2), (-3, -5), (3, -2) and (2, 3).

4. A median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A (4,-6), B (3, -2) and C (5, 2).

5. Find the ratio in which the point P (x, 2), divides the line segment joining the points A (12, 5) and B (4, -3). Also find the value of x.

6. If A (4, 2), B (7, 6) and C (1, 4) are the vertices of a ΔABC and AD is its median, prove that the median AD divides into two triangles of equal areas.

7. If the point A (2, -4) is equidistant from P (3, 8) and Q (-10, y), find the values

of y. Also find distance PQ.

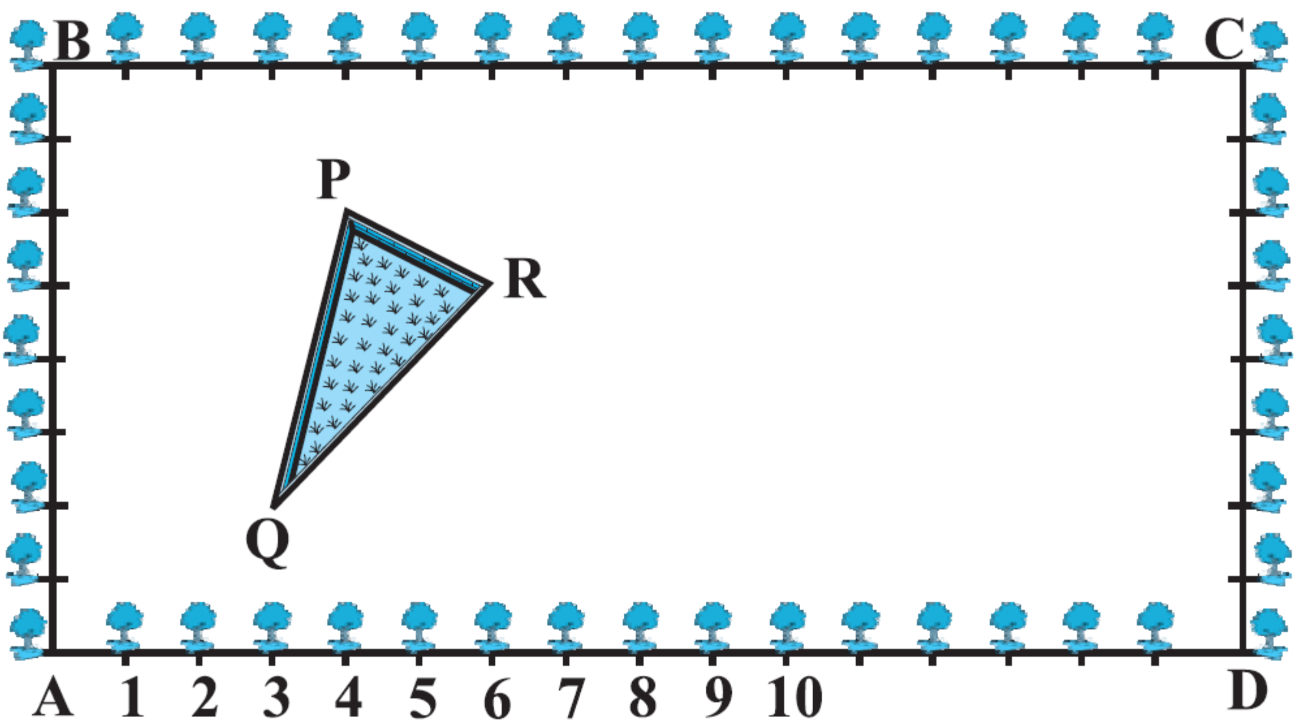
8. The base BC of an equilateral triangle ABC lies on y-axis. The coordinates of point Care (0, -3). The origin is the mid-point of the base. Find the coordinates of the points A and B. Also find the coordinates of another point D such that BACD is a rhombus.

9. Prove that the area of a triangle with vertices (t, t-2), (t + 2, t + 2) and (t + 3, t) is independent of t.

10. The area of a triangle is 5 sq units. Two of its vertices are (2, 1) and (3, -2). If the third vertex is $\left(\frac{7}{2}, y\right)$, find the value of y.

Case Study Qurstions:

1. The Class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Sapling of Gulmohar is planted on the boundary of the plot at a distance of 1m from each other. There is a triangular grassy lawn inside the plot as shown in Fig. The students have to sow seeds of flowering plants on the remaining area of the plot.



i. Considering A as the origin, what are the coordinates of A?

a. (0, 1)
b. (1, 0)
c. (0, 0)
d. (-1, -1)

ii. What are the coordinates of P?

a. (4, 6)
b. (6, 4)
c. (4, 5)
d. (5, 4)

iii. What are the coordinates of R?

a. (6, 5)
b. (5, 6)
c. (6, 0)
d. (7, 4)

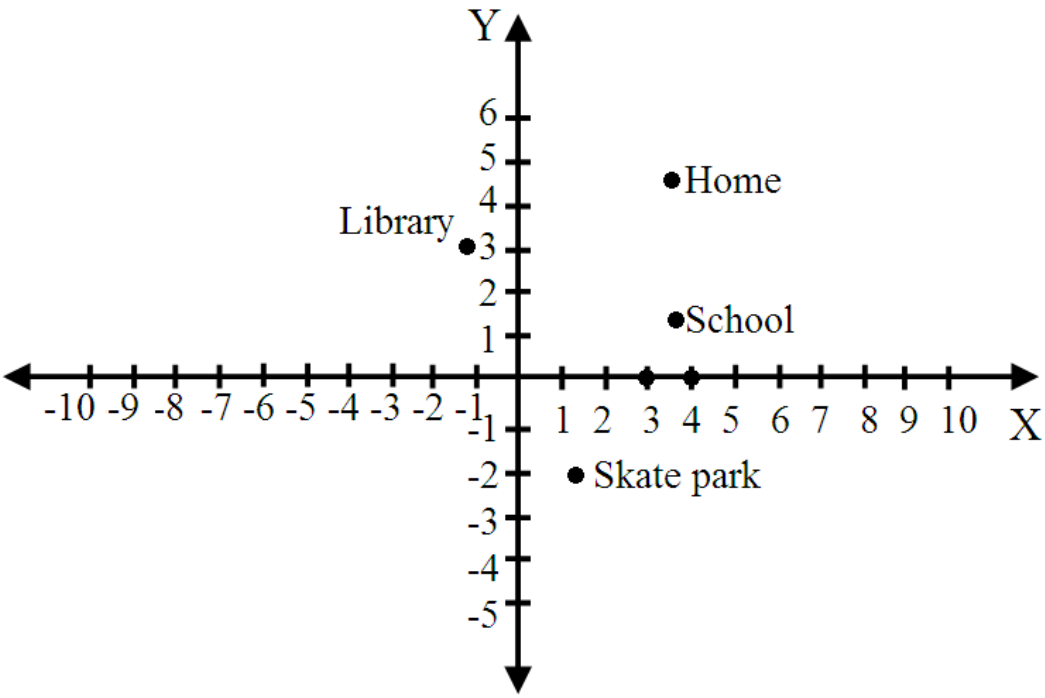
iv. What are the coordinates of D?

a. (16, 0)
b. (0, 0)
c. (0, 16)
d. (16, 1)

v. What are the coordinates of P if D is taken as the origin?

a. (12, 2)
b. (-12, 6)
c. (12, 3)
d. (6, 10)

2. Two brothers Ramesh and Pulkit were at home and have to reach School. Ramesh went to Library first to return a book and then reaches School directly whereas Pulkit went to Skate Park first to meet his friend and then reaches School directly.



i. How far is School from their Home?

a. 5m
b. 3m
c. 2m
d. 4m

ii. What is the extra distance travelled by Ramesh in reaching his School?

a. 4.48 metres
b. 6.48 metres
c. 7.48 metres
d. 8.48 metres

iii. What is the extra distance travelled by Pulkit in reaching his School? (All distances are measured in metres as straight lines).

a. 6.33 metres
b. 7.33 metres
c. 5.33 metres
d. 4.33 metres

iv. The location of the library is:

a. (-1, 3)
b. (1, 3)
c. (3, 1)
d. (3, -1)

v. The location of the Home is:

a. (4, 2)
b. (1, 3)
c. (4, 5)
d. (5, 4)

Assertion Reason Questions-

1. **Directions:** Each of these questions contains two statements: Assertion [A] and Reason [R]. Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes [a], [b], [c] and [d] given below.

   a. A is true, R is true; R is a correct explanation for A.

   b. A is true, R is true; R is not a correct explanation for A.

   c. A is true; R is False.

   d. A is false; R is true.

2. **Directions:** Each of these questions contains two statements: Assertion [A] and Reason [R]. Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes [a], [b], [c] and [d] given below.

   a. A is true, R is true; R is a correct explanation for A.

   b. A is true, R is true; R is not a correct explanation for A.

   c. A is true; R is False.

   d. A is false; R is true.

Answer Key-

Multiple Choice questions-

1. (a) 2:3
2. (d) x = 0, y = ± √7
3. (d) 5√5 units
4. (c) 2:1
5. (d) X-axis and Y-axis
6. (d) 2,2
7. (b) collinear
8. (a) 3 : 4
9. (a) (1,2) or (3,6)
10. (c) (-4, 2)

Very Short Answer :

1. Area of ΔOAB = $\frac{1}{2}$ [0(0 – 1) – 3(0 – 0) + 5(0 – 0)] = 0

⇒ Given points are collinear

2.

$$\text{Centroid of } \Delta PQR = \left(\frac{a+b+c}{3}, \frac{b+c+a}{3}\right)$$

$$\text{Given } \left(\frac{a+b+c}{3}, \frac{b+c+a}{3}\right) = (0, 0)$$

$$\Rightarrow \quad a + b + c = 0$$

3.

$$\text{Length of diagonal} = AB = \sqrt{(5-0)^2 + (0-3)^2} = \sqrt{25+9} = \sqrt{34}$$

4. Since (3, a) lies on the line 2x – 3y = 5

Then 2(3) – 3(a) = 5

$-3a = 5 - 6$

$-3a = -1$

$\Rightarrow a = \frac{1}{3}$

5. Here $x_1 = 0$, $y_1 = 5$, $x_2 = -5$ and $y_2 = 0$)

$$\therefore \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-5-0)^2 + (0-5)^2}$$

$$= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2} \text{ units}$$

6. Here $x_1 = -6$, $y_1 = 8$

$x_2 = 0$, $y_2 = 0$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{[0-(-6)^2] + (0-8)^2} = \sqrt{(6)^2 + (-8)^2} = \sqrt{36+64}$$

$$= \sqrt{100} = 10 \text{ units}$$

7. Using distance formula

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \text{distance given}$$

$$\sqrt{(4-1)^2 + (k-0)^2} = 5$$

$$9 + k^2 = 25 \quad \Rightarrow \quad k^2 = 16$$

$$\Rightarrow \quad k = \pm 4$$

8. Points A, B and C are collinear

$\Rightarrow 1(0 - b) + 0 (b - 2) + a(2 - 0) = 0$

$\Rightarrow -b + 2a = 0$ or $2a = b$

9. In Fig. 6.6, let the point P(-1, 6) divides the line joining A(-3, 10) and B (6, -8) in the ratio k : 1

then, the coordinates of P are $\left(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1}\right)$

But, the coordinates of P are $(-1, 6)$

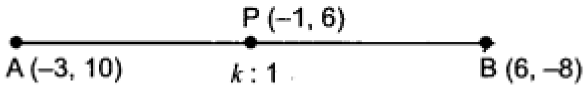


Fig. 6.6

$$\therefore \quad \frac{6k-3}{k+1} = -1 \quad \Rightarrow \quad 6k - 3 = -k - 1$$
$$\Rightarrow \quad 6k + k = 3 - 1 \quad \Rightarrow \quad 7k = 2$$
$$\Rightarrow \quad k = \frac{2}{7}$$

Hence, the point P divides AB in the ratio 2 : 7.

10.

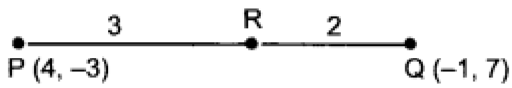


Fig. 6.7

$$\frac{PQ}{PR} = \frac{5}{3} \quad \Rightarrow \quad \frac{PQ - PR}{PR} = \frac{5-3}{3}$$
$$\Rightarrow \quad \frac{RQ}{PR} = \frac{2}{3}$$

i.e., R divides PQ in the ratio 3 : 2

Abscissa of $R = \dfrac{3\times(-1)+2\times 4}{3+2} = \dfrac{-3+8}{5} = 1$

Short Answer :

1. Let the required point be (x, 0).

Since, (x, 0) is equidistant from the points (-3, 4) and (2, 5).

$$\therefore \quad \sqrt{(-3-x)^2 + (4-0)^2} = \sqrt{(2-x)^2 + (5-0)^2}$$
$$\Rightarrow \quad \sqrt{9 + x^2 + 6x + 16} = \sqrt{4 + x^2 - 4x + 25}$$
$$\Rightarrow \quad x^2 + 6x + 25 = x^2 - 4x + 29 \quad \Rightarrow \quad 10x = 4 \quad \text{or} \quad x = \frac{4}{10} = \frac{2}{5}$$
$$\therefore \quad \text{Required point is } \left(\frac{2}{5},\ 0\right).$$

2.

$$\text{Distance between the given points} = \sqrt{(x-2)^2 + (5+3)^2}$$

$$\Rightarrow \quad 10 = \sqrt{x^2 + 4 - 4x + 64}$$

$$\Rightarrow \quad 100 = x^2 - 4x + 68$$

$$\Rightarrow \quad x^2 - 4x - 32 = 0$$

$$\Rightarrow \quad x^2 - 8x + 4x - 32 = 0$$

$$\Rightarrow \quad (x-8)(x+4) = 0 \quad \Rightarrow \quad x = 8, -4$$

3.

$$\text{Distance between the given points} = \sqrt{(0 - 10\cos 30°)^2 + (10\cos 60° - 0)^2}$$

$$= \sqrt{100\cos^2 30° + 100\cos^2 60°}$$

$$= \sqrt{100\left[\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right]} = \sqrt{100\left(\frac{3}{4} + \frac{1}{4}\right)} = \sqrt{100} = 10 \text{ units}$$

4.

$$\text{Coordinates of the mid-point of } BC = \left(\frac{1+5}{2}, \frac{-1+1}{2}\right) = (3, 0)$$

$$\therefore \text{ Length of the median through } A = \sqrt{(3+1)^2 + (0-3)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ units.}$$

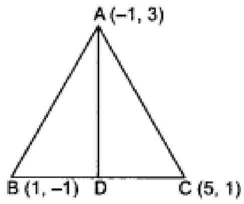


Fig. 6.8

5. Let the required ratio be λ : 1

$$\text{Then, the point of division is } \left(\frac{5\lambda + 3}{\lambda + 1}, \frac{3\lambda - 6}{\lambda + 1}\right)$$

Given that this point lies on the x-axis

$$\therefore \quad \frac{3\lambda - 6}{\lambda + 1} = 0 \quad \text{or} \quad 3\lambda = 6 \quad \text{or} \quad \lambda = 2$$

Thus, the required ratio is 2 : 1.

6. Points of trisection of line segment AB are given by

$$= \left(\frac{2\times1+1\times7}{3}, \frac{2\times(-5)+1\times(-2)}{3}\right) \text{ and } \left(\frac{1\times1+2\times7}{3}, \frac{1\times(-5)+2\times(-2)}{3}\right)$$

$$= \left(\frac{9}{3}, \frac{-12}{3}\right) \text{and} \left(\frac{15}{3}, \frac{-9}{3}\right) \text{ or } (3, -4) \text{ and } (5, -3)$$

∴ Given statement is true.

7.

$$AB = \sqrt{(2+2)^2+0} = \sqrt{16} = 4$$
$$BC = \sqrt{(0-2)^2+(2-0)^2} = \sqrt{8} = 2\sqrt{2}$$
$$CA = \sqrt{(-2-0)^2+(0-2)^2} = \sqrt{8} = 2\sqrt{2}$$
$$PQ = \sqrt{(4+4)^2+0} = \sqrt{64} = 8$$
$$QR = \sqrt{(0-4)^2+(4-0)^2} = \sqrt{32} = 4\sqrt{2}$$
$$RP = \sqrt{(-4-0)^2+(0-4)^2} = \sqrt{32} = 4\sqrt{2}$$
$$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{1}{2} \quad \Rightarrow \quad \Delta ABC \sim \Delta PQR$$

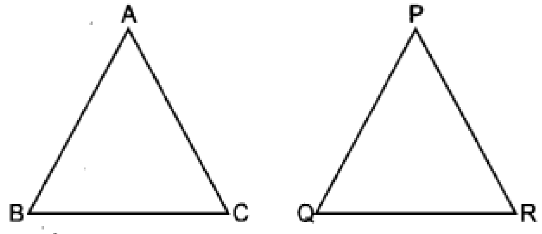


Fig. 6.9

8. The point P (0, 2) lies on y-axis

$$\text{Also,} \quad AP = \sqrt{(0+1)^2+(2-1)^2} = \sqrt{2}$$
$$BP = \sqrt{(0-3)^2+(2-3)^2} = \sqrt{9+1} = \sqrt{10}$$

AP ≠ BP

∴ P(0, 2) does not lie on the perpendicular bisector of AB. So, given statement is false.

9. Let A (1, 5), B (2, 3) and C (-2, -11) be the given points. Then we have

$$AB = \sqrt{(2-1)^2+(3-5)^2} = \sqrt{1+4} = \sqrt{5}$$
$$BC = \sqrt{(-2-2)^2+(-11-3)^2} = \sqrt{16+196} = \sqrt{4\times 53} = 2\sqrt{53}$$
$$AC = \sqrt{(-2-1)^2+(-11-5)^2} = \sqrt{9+256} = \sqrt{265}$$

Clearly, AB + BC ≠ AC

∴ A, B, C are not collinear.

10. (i) Let two given points be A (-5, 7) and B (-1, 3).

Thus, we have $x_1 = -5$ and $x_2 = -1$

$y_1 = 7$ and $y_2 = 3$

$$\therefore \quad AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$\Rightarrow \quad AB = \sqrt{(-1+5)^2+(3-7)^2} = \sqrt{(4)^2+(-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \text{ units.}$$

Long Answer :

1. Let the given points be

A $(x_1, y_1) = (7, -2)$, B $(x_2, Y_2) = (5, 1)$ and C $(x_3, y_3) = (3, k)$

Since these points are collinear therefore area (ΔABC) = 0

$\Rightarrow 12\ [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow x1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\Rightarrow 7(1 - k) + 5(k + 2) + 3(-2 -1) = 0$

$\Rightarrow 7 - 7k + 5k + 10 - 9 = 0$

$\Rightarrow -2k + 8 = 0$

$\Rightarrow 2k = 8$

$\Rightarrow k = 4$

Hence, given points are collinear for k = 4.

2. Let A $(x_1, y_1) = (0, -1)$, B $(x_2, y_2) = (2, 1)$, C $(x_3, y_3) = (0, 3)$ be the vertices of ΔABC.

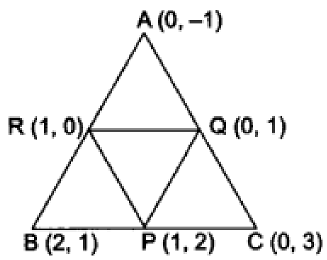
Now, let P, Q, R be the mid-points of BC, CA and AB, respectively.

So, coordinates of P, Q, R are

$$P = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$$

$$Q = \left(\frac{0+0}{2}, \frac{3-1}{2}\right) = (0, 1)$$

$$R = \left(\frac{2+0}{2}, \frac{1-1}{2}\right) = (1, 0)$$



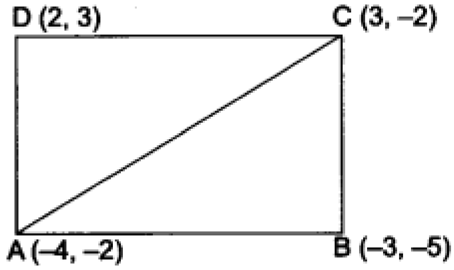
Therefore, $ar\ (\Delta PQR) = \frac{1}{2}[1(1-0) + 0(0-2) + 1(2-1)] = \frac{1}{2}\ (1 + 1) = 1$ sq. unit

Now, $ar(\Delta ABC) = \frac{1}{2}[0(1-3) + 2\ (3 + 1) + 0\ (-1 -1)]$

$$= \frac{1}{2}[0 + 8 + 0] = \frac{8}{2} = 4 \text{ sq. units}$$

Ratio of ar (ΔPQR) to the ar (ΔABC) = 1 : 4.

3.



Let A(4, -2), B(-3, -5), C(3, -2) and D(2, 3) be the vertices of the quadrilateral ABCD.

Now, area of quadrilateral ABCD

= area of ΔABC + area of ΔADC

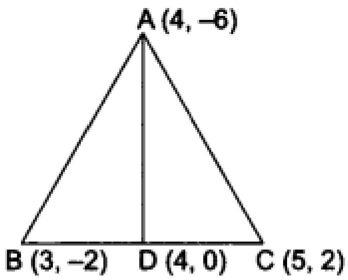
$$= \frac{1}{2}[-4(-5+2) - 3(-2+2) + 3(-2+5)] + \frac{1}{2}[-4(-2-3) + 3(3+2) + 2(-2+2)$$

$$= \frac{1}{2}[12 - 0 + 9] + \frac{1}{2}[20 + 15 + 0]$$

$$\frac{1}{2}[21 + 35] = \frac{1}{2} \times 56 = 28 \text{ sq. units.}$$

4. Since AD is the median of ΔABC, therefore, D is the mid-point of BC.

Coordinates of D are $\left(\frac{3+5}{2}, \frac{-2+2}{2}\right)$ *i.e.*, (4, 0)



Now, area of ΔABD

$$= \frac{1}{2}[4(-2-0) + 3(0+6) + 4(-6+2)]$$

$$= \frac{1}{2}(-8 + 18 - 16) = \frac{1}{2} \times (-6) = -3$$

Since area is a measure, it cannot be negative.

Therefore, $ar(\Delta ABD) = 3$ sq. units

and area of $\Delta ADC = \frac{1}{2}[4(0-2) + 4(2+6) + 5(-6-0)$

$$= \frac{1}{2}(-8 + 32 - 30)$$

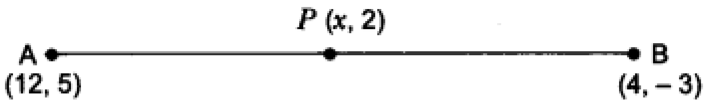
$= \frac{1}{2}(-6) = -3$, which cannot be negative.

$\therefore$ $ar(\Delta ADC) = 3$ sq. units

Here, $ar(\Delta ABD) = ar(\Delta ADC)$

Hence, the median divides it into two triangles of equal areas.

5.



Let the ratio in which point P divides the line segment be k:1.

Then, coordinates of P : $\left(\dfrac{4k+12}{k+1}, \dfrac{-3k+5}{k+1}\right)$

Given, the coordinates of P as $(x, 2)$

$$\therefore \quad \frac{4k+12}{k+1} = x \qquad \ldots(i)$$

$$\text{and} \quad \frac{-3k+5}{k+1} = 2 \qquad \ldots(ii)$$

$$-3k + 5 = 2k + 2$$

$$5k = 3 \quad \Rightarrow \quad k = \frac{3}{5}$$

Putting the value of k in (i), we have

$$\frac{4\times\frac{3}{5}+12}{\frac{3}{5}+1} = x \quad \Rightarrow \quad \frac{12+60}{3+5} = x$$

$$x = \frac{72}{8} \quad \Rightarrow \quad x = 9$$

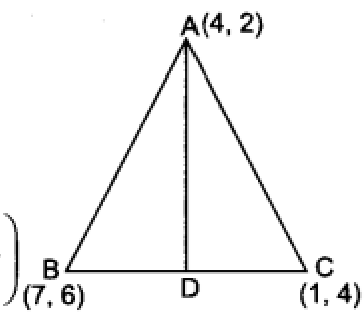
The ratio in which p divides the line segment is $\frac{3}{5}$, i.e., 3 : 5.

6. Given: AD is the median on BC.

$\Rightarrow$ BD = DC

The coordinates of midpoint D are given by.

$$\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right) \qquad i.e., \left(\frac{1+7}{2}, \frac{4+6}{2}\right)$$



Coordinates of D are (4, 5).

Now, Area of triangle $ABD = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$$= \frac{1}{2}|4(6-5) + 7(5-2) + 4(2-6)| = \frac{1}{2}|4 + 21 - 16| = \frac{9}{2} \text{ sq. units}$$

$$\text{Area of } \Delta ACD = \frac{1}{2}|4(4-5) + 1(5-2) + 4(2-4)|$$

$$= \frac{1}{2}\ |-4+3-8| = \frac{1}{2}\ |-9| = \frac{9}{2} \text{ sq. units}$$

Hence, AD divides ΔABC into two equal areas.

7. Given points are A(2, 4), P(3, 8) and Q(-10, y)

According to the question,

PA = QA

$$\sqrt{(2-3)^2 + (-4-8)^2} = \sqrt{(2+10)^2 + (-4-y)^2}$$

$$\sqrt{(-1)^2 + (-12)^2} = \sqrt{(12)^2 + (4+y)^2}$$

$$\sqrt{1+144} = \sqrt{144+16+y^2+8y}$$

$$\sqrt{145} = \sqrt{160+y^2+8y}$$

On squaring both sides, we get

$$145 = 160 + y^2 + 8y$$

$$y^2 + 8y + 160 - 145 = 0$$

$$y^2 + 8y + 15 = 0$$

$$y^2 + 5y + 3y + 15 = 0$$

$$y(y+5) + 3(y+5) = 0$$

$$\Rightarrow \quad (y+5)(y+3) = 0$$

$$\Rightarrow \quad y + 5 = 0 \quad \Rightarrow \quad y = -5$$

$$\text{and} \quad y + 3 = 0 \quad \Rightarrow \quad y = -3$$

$$\therefore \quad y = -3, -5$$

$$\text{Now,} \quad PQ = \sqrt{(-10-3)^2 + (y-8)^2}$$

$$\text{For } y = -3 \quad PQ = \sqrt{(-13)^2 + (-3-8)^2} = \sqrt{169+121} = \sqrt{290} \text{ units}$$

$$\text{and for } y = -5 \quad PQ = \sqrt{(-13)^2 + (-5-8)^2} = \sqrt{169+169} = \sqrt{338} \text{ units}$$

Hence, values of y are -3 and -5, $PQ = \sqrt{290}$ and $\sqrt{338}$ units.

8. ∵ O is the mid-point of the base BC.

∴ Coordinates of point B are (0, 3). So,

BC = 6 units Let the coordinates of point A be (x, 0).

Using distance formula,

$$AB = \sqrt{(0-x)^2+(3-0)^2} = \sqrt{x^2+9}$$

$$BC = \sqrt{(0-0)^2+(-3-3)^2} = \sqrt{36}$$

Also, $AB = BC$ ($\because$ ΔABC is an equilateral triangle)

$$\sqrt{x^2+9} = \sqrt{36}$$

$$x^2 + 9 = 36$$

$$x^2 = 27 \quad \Rightarrow \quad x^2 - 27 = 0$$

$$x^2 - (3\sqrt{3})^2 = 0 \quad \Rightarrow \quad (x + 3\sqrt{3})(x - 3\sqrt{3}) = 0$$

$$x = -3\sqrt{3} \text{ or } x = 3\sqrt{3}$$

$$\Rightarrow \quad x = \pm\, 3\sqrt{3}$$

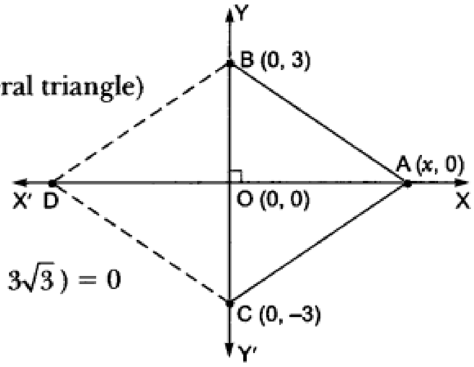


Fig. 6.30

∴ Coordinates of point A = (x, 0) = ($3\sqrt{3}$, 0)

Since BACD is a rhombus.

∴ AB = AC = CD = DB

∴ Coordinates of point D = ($-3\sqrt{3}$, 0).

9. Area of a triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of the triangle = $\frac{1}{2}[t + 2 - t) + (t + 2)(t - t + 2) + (t + 3)(t - 2 - t - 2)]$

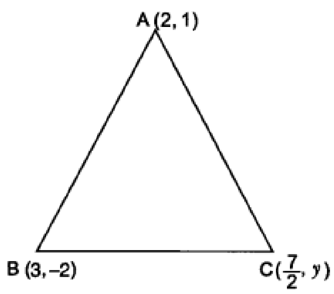
$= \frac{1}{2}[2t + 2t + 4 - 4t - 12]$

= 4 sq. units

which is independent of t.

Hence proved.

10.



Given: ar(ΔABC) = 5 sq. units

$$\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|=5$$

$$\Rightarrow \quad \frac{1}{2}\left|2(-2-y)+3(y-1)+\frac{7}{2}(1+2)\right|=5$$

$$\Rightarrow \quad -4-2y+3y-3+\frac{7}{2}+7=10$$

$$\Rightarrow \quad y+\frac{7}{2}=10 \quad \Rightarrow \quad y=10-\frac{7}{2}$$

$$\Rightarrow \quad y=\frac{13}{2}$$

Case Study Answer-

1. **Answer :**

It can be observed that the coordinates of point P, Q and R are (4, 6), (3, 2), and (6, 5) respectively.

i	c	(0, 0)
ii	a	(4, 6)
iii	a	(6, 5)
iv	a	(16, 0)
v	b	(-12, 6)

2. **Answer :**

i. (b) Distance between home and school, $\mathrm{HS}=\sqrt{(4-4)^2+(3-5)^2}=3\mathrm{m}$

ii. (c) Now, $\mathrm{HL}=\sqrt{(-1-4^2)+(3-5)^2}=\sqrt{25+4}=\sqrt{29}$

$$\mathrm{LS}=\sqrt{[4-(-1)]^2+(2-3)^2}=\sqrt{25+1}=\sqrt{26}$$

Thus, $\mathrm{HL}+\mathrm{LS}=\sqrt{29}+\sqrt{26}=10.48\mathrm{m}$

So, extra distance covered by ramesh is $=\mathrm{HL}+\mathrm{LS}-\mathrm{HS}=10.48-3=7.48\mathrm{m}$

iii. (d) Now, $\mathrm{HP}=\sqrt{(3-4)^2+(0-5)^2}=\sqrt{1+25}=\sqrt{26}$

$$\mathrm{PS}=\sqrt{[4-3]^2+(2-0)^2}=\sqrt{1+4}=\sqrt{5}$$

Thus, $\mathrm{HP}+\mathrm{PS}=\sqrt{26}+\sqrt{5}=7.33\mathrm{m}$

So, extra distance covered by pulkit is $=\mathrm{HP}+\mathrm{PS}-\mathrm{HS}=7.33-3=4.33\mathrm{m}$

iv. (a) (-1, 3)
v. (c) (4, 5)

Assertion Reason Answer-

1. (a) A is true, R is true; R is a correct explanation for A.
2. (d) A is false; R is true.