MATHEMATICS

Chapter 11: Constructions

Constructions

To Construct an Angle Equal to a Given Angle

Given: Any ∠POQ and a point A

Required: To construct an angle at A equal to ∠POQ

Steps of Construction:

i. With O as centre and any (suitable) radius, draw an arc to meet OP at R and OQ at S.

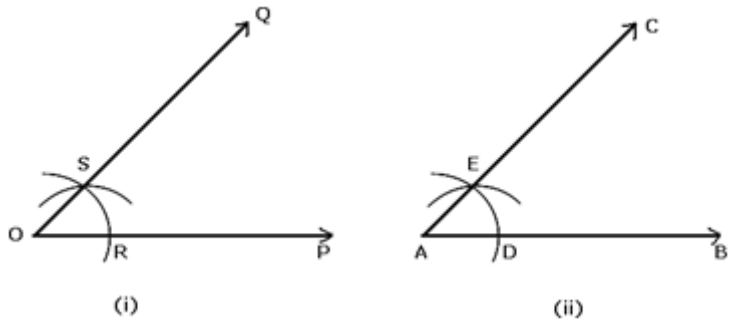
ii. Through A draw a line AB.

iii. Taking A as centre and same radius (as in step 1), draw an arc to meet AB at D.

iv. Measure the segment RS with compasses.

v. With D as centre and radius equal to RS, draw an arc to meet the previous arc at E.

vi. Join AE and produce it to C, then ∠BAC is the required angle equal to ∠POQ.



Linear Pair axiom

- If a ray stands on a line then the adjacent angles form a linear pair of angles.
- If two angles form a linear pair, then uncommon arms of both the angles form a straight line.

To Bisect a Given Angle

Given: Any ∠POQ

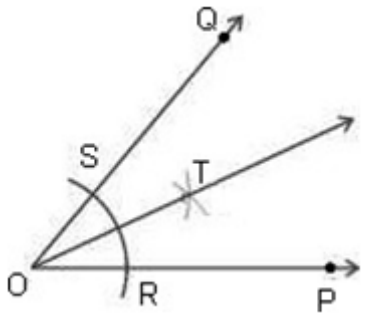
Required: To bisect ∠POQ.

Steps of Construction:

i. With O as centre and any (suitable) radius, draw an arc to meet OP at R and OQ at S.

ii. With R as centre and radius more than half of RS, draw an arc. Also, with S as centre and same radius draw another arc to meet the previous arc at T.

iii. Join OT and produce it, then OT is the required bisector of ∠POQ.



To Construct some Specific Angles

To construct an angle of 60°

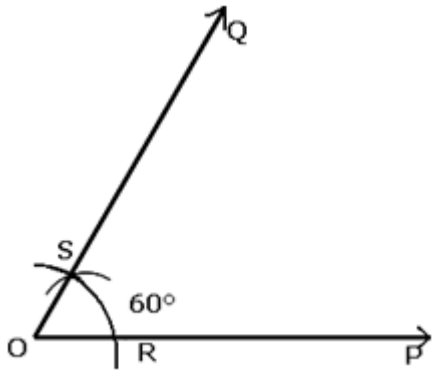
Steps of Construction:

i. Draw any line OP.

ii. With O as centre and any suitable radius, draw an arc to meet OP at R.

iii. With R as centre and same radius (as in step 2), draw an arc to meet the previous arc at S.

iv. Join OS and produce it to Q, then ∠POQ = 60°.

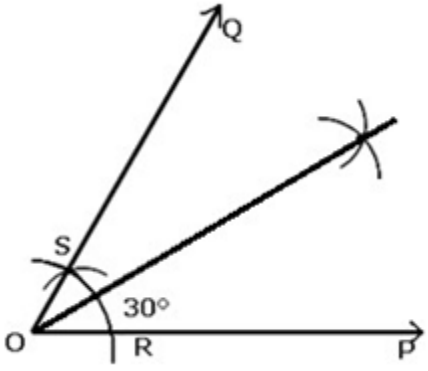


To construct an angle of 30°

Steps of Construction

i. Construct ∠POQ = 60°.

ii. Bisect ∠POQ. Let OT be the bisector of ∠POQ, then ∠POT = 30°



To construct an angle of 120°

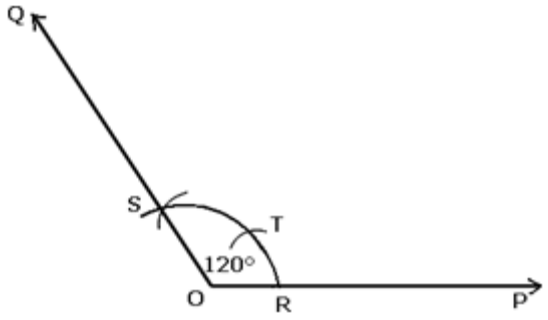
Steps of Construction:

i. Draw any line OP.

ii. With O as centre and any suitable radius, draw an arc to meet OP at R.

iii. With R as centre and same radius (as in step 2), draw an arc to meet the previous arc at T. With T as centre and same radius, draw another arc to cut the first arc at S.

iv. Join OS and produce it to Q, then ∠POQ = 120°.



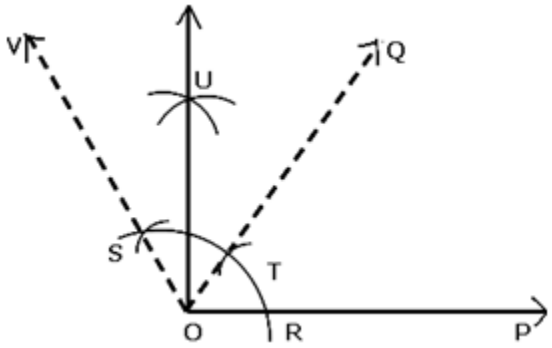
To construct an angle of 90°

Steps of Construction:

i. Construct ∠POQ = 60°

ii. Construct ∠POV = 120°.

iii. Bisect ∠QOV. Let OU be the bisector of ∠QOV, then ∠POU = 90°.



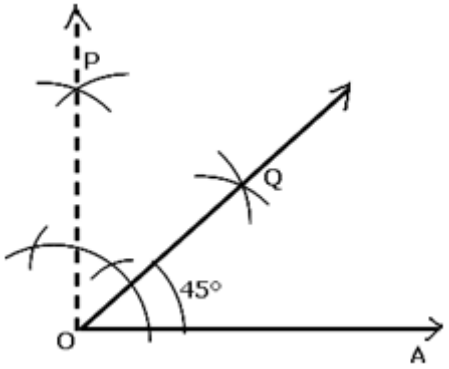
To construct an angle of 45°

Steps of Construction:

i. Construct ∠AOP = 90°.

ii. Bisect ∠AOP.

iii. Let OQ be the bisector of ∠AOP, then ∠AOQ = 45°



To Draw a Perpendicular Bisector of a Line Segment

Given: Any line segment PQ.

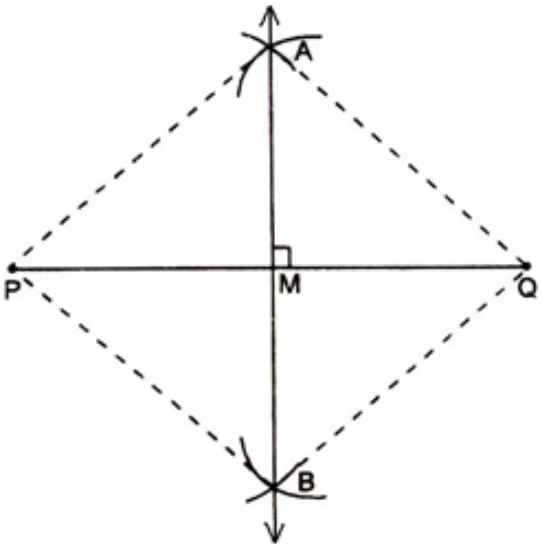
Required: To draw a perpendicular bisector of line segment PQ.

Steps of Construction:

i. With P as centre, take a length greater than half of PQ and draw arcs one on each side of PQ.

ii. With Q as centre and same radius (as in step 1), draw two arcs on each side of PQ cutting the previous arcs at A and B.

iii. Join AB to meet PQ at M, then AB bisects PQ at M, and is perpendicular to PQ, Thus, AB is the required perpendicular bisector of PQ.



Properties of a Perpendicular Bisector

- It divides AB into two equal halves or bisects it.
- It makes right angles with (or is perpendicular to) AB.
- Every point in the perpendicular bisector is equidistant from point A and B.

While working with practical geometry, you will often find the application of perpendicular bisectors; say when you are asked to draw an isosceles triangle, or when you have to determine the centre of a circle, etc. Below are the steps to construct a perpendicular

bisector of a line using a compass and a ruler.

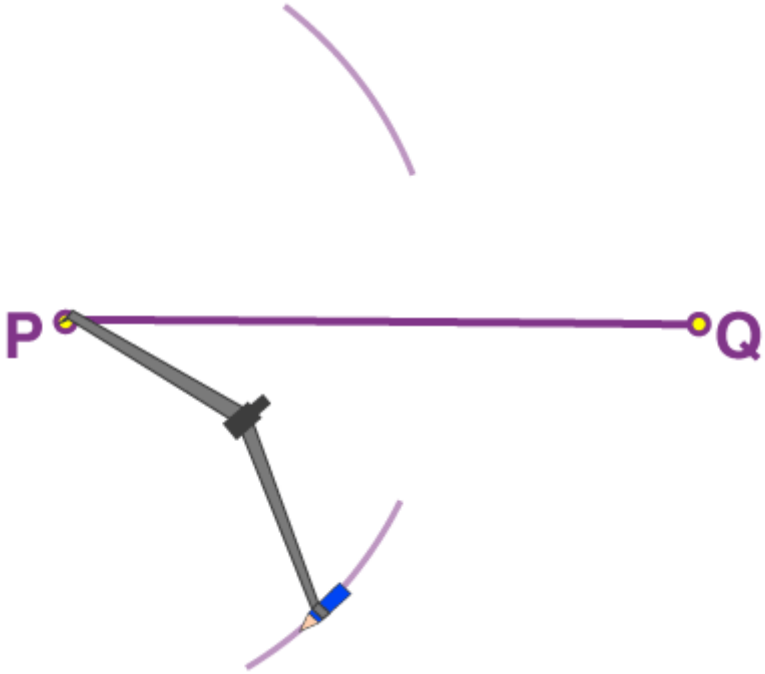
How to Construct a Perpendicular Bisector?

You will require a ruler and compasses. The steps for the construction of a perpendicular bisector of a line segment are:

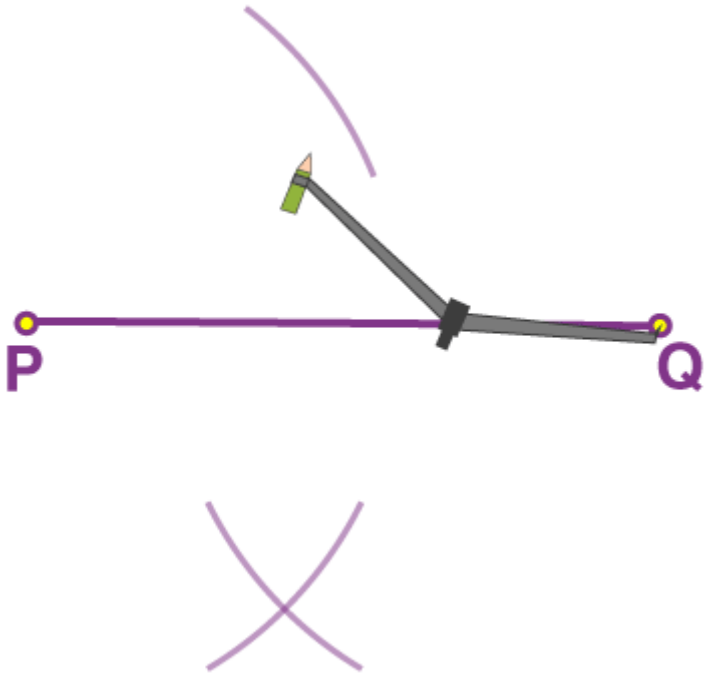
Step 1: Draw a line segment PQ.

Step 2: Adjust the compass with a length of a little more than half of the length of PQ.

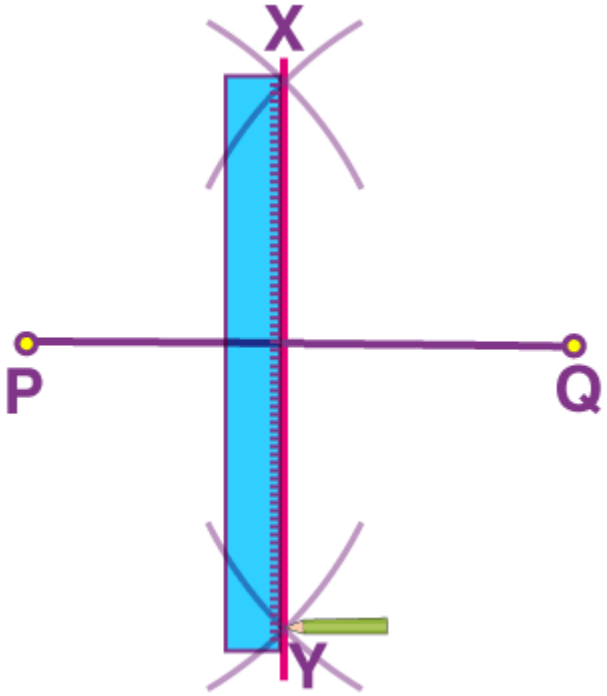
Step 3: Place the compass pointer at point P and draw arcs above and below the line.



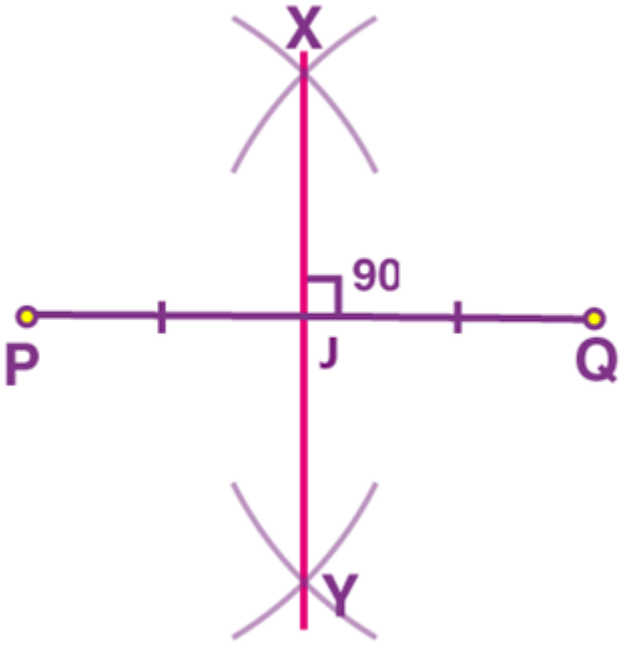
Step 4: Keeping the same length in the compass, place the compass pointer at point Q. Similarly, draw two arcs above and below the line keeping the compass pointer at Q.



Step 5: Mark the points where the opposite arcs cross as X and Y.



Step 6: Using a ruler, draw a line passing across X and Y.



The perpendicular bisector bisects PQ at a point J, that is, the length PJ is equal to JQ. And the angle between the two lines is 90 degrees.

Construction of a Triangle, given its Base, sum of the other two sides and one Base Angle

To construct ΔABC in which base BC, ∠B and sum AC + AB of other two sides are given.

Steps of construction:

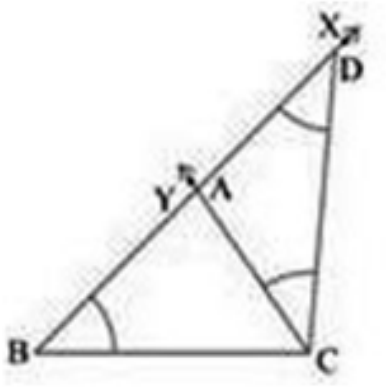
i. Draw the base BC and at the point B, make an angle, say XBC equal to the given angle.

ii. Cut a line segment BD = AC + AB from the ray BX.

iii. Join DC and make angle DCY equal to angle BDC.

iv. Let CY intersect BX at A.

v. ABC is the required triangle.



Alternate Method

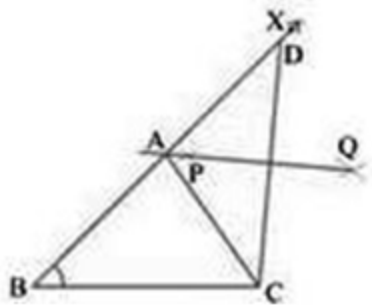
Steps of construction:

i. Draw the base BC and at the point B, make an angle, say XBC equal to the given angle.

ii. Cut a line segment BD = AC + AB from the ray BX.

iii. Draw perpendicular bisector PQ of CD to intersect BD at a point A. Join AC.

ABC is the required triangle.



Construction of a Triangle, given its Base, difference of the other two sides and one Base

To construct ΔABC when the base BC, a base angle B and the difference of other two sides AB - AC or AC - AB are given.

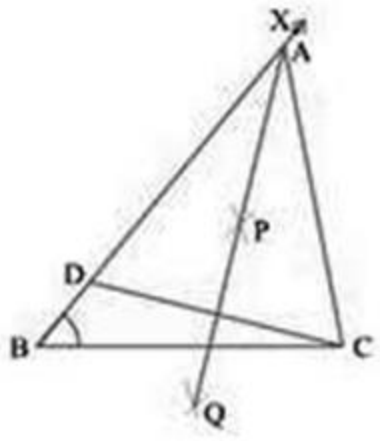
Case 1: When AB > AC and AB - AC is given

Steps of construction:

i. Draw the base BC and at point B make an angle say XBC equal to the given angle.

ii. Cut the line segment BD equal to AB - AC from ray BX.

iii. Join DC and draw the perpendicular bisector, say PQ of DC. Let it intersect BX at a point A. Join AC Then, ABC is the required triangle.



Case 1: When AB < AC and AC - AB is given Steps of Construction:

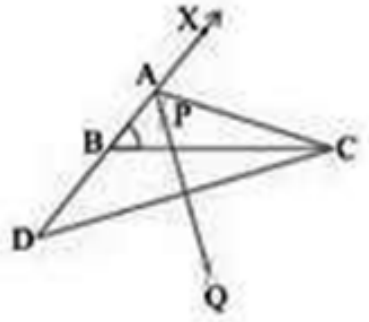
i. Draw the base BC and at point B make an angle say XBC equal to the given angle.

ii. Cut a line segment BD equal to AC - AB from the line BX extended on opposite side of line segment BC.

iii. Join DC and draw the perpendicular bisector, say PQ of DC.

iv. Let PQ intersect BX at A. Join AC.

Then, ABC is the required triangle.



Construction of a Triangle of given Perimeter and Base Angles

To construct a triangle ABC, when its perimeter, AB + BC + CA, and two base angles, ∠B and ∠C, are given.

Steps of Construction:

i. Draw a line segment, say XY = BC + CA + AB.

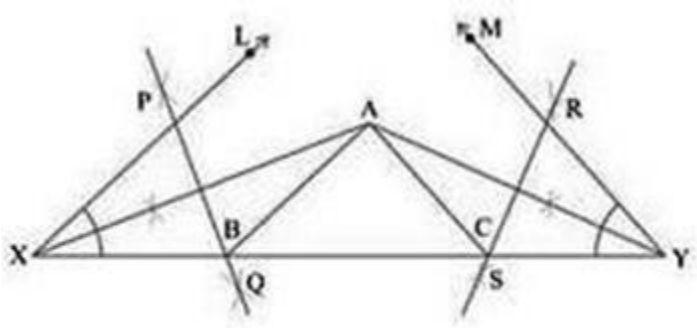
ii. Construct ∠LXY = ∠B and ∠MYX = ∠C.

iii. Draw the bisectors of ∠LXY and ∠MYX. Let these bisectors intersect at point A

iv. Draw a perpendicular bisector PQ of AX and RS of AY.

v. Let PQ intersect XY at B and RS intersect XY at C.

vi. Join AB and AC. Then, ABC is the required triangle.


L
M
P
A
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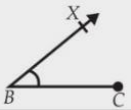
CHAPTER : 11 CONSTRUCTIONS

Constructions

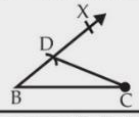
Triangle

Given: Base, Angles, difference of two sides

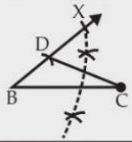
Step 1 : Draw BC = base and make given angle.



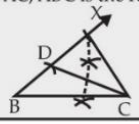
Step 2 : Cut BD = difference of sides join DC



Step 3 : Draw perpendicular bisector DC and let it intersect BX. Name it as A.

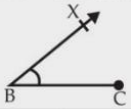


Step 4 : Join AC, ABC is the required triangle.

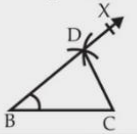


Given : Base, Angles sum of two sides

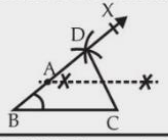
Step 1 : Draw BC = base and make given angle.



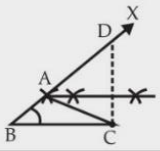
Step 2 : Cut BD equal to sum of sides, join DC



Step 3 : Draw perpendicular bisector of DC and let it intersect BD. Name it as A.

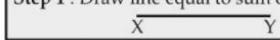


Step 4 : Join AC, ABC is the required triangle.



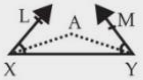
Given : Perimeter and two base angles

Step 1 : Draw line equal to sum of sides = XY



Step 2 : Draw given two angles at X & Y

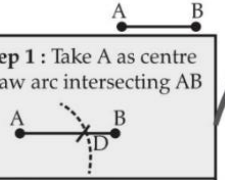
Step 3 : Bisect ∠LXY and ∠MYX, let the bisectors meet at A.



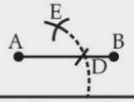
Step 4 : Draw perpendicular bisectors of AX and AY. Extend them to intersect XY. ABC is the required triangle.

60° Angle

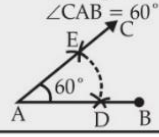
Step 1 : Take A as centre draw arc intersecting AB



Step 2 : With D as centre and same radius, draw arc intersecting same arc.

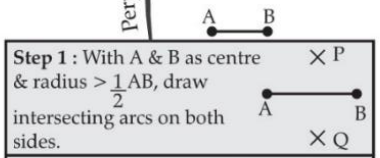


Step 3 : Draw AC passing through E, ∠CAB = 60°

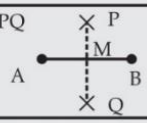


Perpendicular bisector

Step 1 : With A & B as centre & radius > $\frac{1}{2}$AB, draw intersecting arcs on both sides.

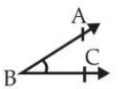


Step 2 : Join PQ

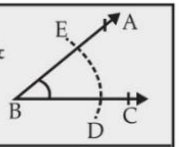


Step 3 : PQ intersect AB at a point M to from perpendicular bisector

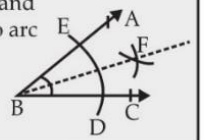
Angle bisector



Step 1 : Take B as centre, draw arcs intersecting AB & BC (of any radius)



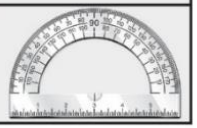
Step 2 : D & E as centres and radius > $\frac{1}{2}$ DE, draw two arc and join B to it



Step 3 : BF is the required angle bisector

Requirements

Geometry box

1. Scale	
2. Pair of set-squares	
3. Pair of dividers	
4. Compass	
4. Protractor	

Geometrical construction

Process of drawing geometrical figure

Important Questions

Multiple Choice questions-

Question 1. If a, b and c are the lengths of the three sides of a triangle, then which of the following is true?

(a) $a + b < c$

(b) $a – b < c$

(c) $a + b = c$

Question 2. With the help of a ruler and compasses, which of the following is not possible to construct?

(a) 70°

(b) 60°

(c) 135°

Question 3. Which of the following sets of angles can be the angles of a triangle?

(a) 30°, 60°, 80°

(b) 40°, 60°, 70°

(c) 50°, 30°, 100°

Question 4. The construction of the triangle ABC is possible if it is given that BC = 4cm, ∠C = 60° and the difference of AB and AC is

(a) 3.5cm

(b) 4.5cm

(c) 3cm

(d) 2.5cm

Question 5. Which of the following can be the length of BC required to construct the triangle ABC such that AC = 7.4cm and AB = 5cm?

(a) 3.5cm

(b) 2.1cm

(c) 4.7cm

Question 6. If we want to construct a triangle, given its perimeter, then we need to know:

(a) Sum of two sides of triangle

(b) Difference between two sides of triangle

(c) One base angles

(d) Two base angles

Question 7. To construct a bisector of a given angle, we need:

(a) A ruler

(b) A compass

(c) A protractor

(d) Both ruler and compass

Question 8. Which of the following set of lengths can be the sides of a triangle?

(a) 2cm, 4cm, 1.9cm

(b) 1.6cm, 3.7cm, 5.3cm

(c) 5.5cm, 6.5cm, 8.9cm

(d) None of the above

Question 9. Which of these angles cannot be constructed using ruler and compasses?

(a) 120

(b) 60

(c) 140

(d) 135

Question 10. Which of the following angles can be constructed using ruler and compasses?

(a) 35

(b) 45

(c) 95

(d) 55

Very Short:

1. Draw a line segment AB = 8cm. Draw $\frac{1}{3}$ part of it. Measure the length of $\frac{1}{3}$ part of AB.

2. Why we cannot construct a ΔABC, if ∠A = 60°, AB = 6cm and AC + BC = 5cm but construction of ΔABC is possible if ∠A = 60°, AB = 6cm and AC - BC = 5cm?

3. Construct an angle of 90° at the initial point of the given ray.

4. Draw a straight angle. Using compass bisect it. Name the angles obtained.

5. Draw any reflex angle. Bisect it using compass. Name the angles so obtained.

Short Questions:

1. Construct a triangle whose sides are in the ratio 2 : 3 : 4 and whose perimeter is 18cm.
2. Construct a? ABC with BC = 8cm? B = 45° and AB - AC = 3.1cm.
3. Construct a ΔABC such that BC = 3.2cm, ∠B = 45° and AC – AB = 2.1cm.
4. Draw a line segment QR = 5 cm. Construct perpendiculars at point Q and R to it. Name them as QX and RY respectively. Are they both parallel?
5. Construct an isosceles triangle whose two equal sides measure 6cm each and whose base is 5cm. Draw the perpendicular bisector of its base and show that it passes through the opposite vertex.

Long Questions:

1. Construct a triangle ABC in which BC = 4.7cm, AB + AC = 8.2cm and ∠C = 60°
2. Construct ΔXYZ, if its perimeter is 14cm, one side of length 5cm and ∠X = 45°
3. To construct a triangle, with perimeter 10cm and base angles 60° and 45°
4. Construct an equilateral triangle whose altitude is 6cm long
5. Construct a rhombus whose diagonals are 8 cm and 6 cm long. Measure the length of each side of the rhombus

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: a, b and c are the lengths of three sides of a triangle, then a+b > c.

Reason: The sum of two sides of a triangle is always greater than the third side.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: The side lengths 4cm, 4cm and 4cm can be sides of equilateral triangle.

Reason: Equilateral triangle has all its three sides equal.

Answer Key:

MCQ:

1. (b) a - b < c
2. (a) 70°
3. (c) 50°, 30°, 100°
4. (b) 4.5cm
5. (b) 2.1cm
6. (c) One base angles
7. (d) Both ruler and compass
8. (c) 5.5cm, 6.5cm, 8.9cm
9. (c) 140
10. (b) 45

Very Short Answer:

1.

Steps of Construction:

1. Draw a line segment AB = 8cm.
2. Draw its perpendicular bisector and let it intersect AB in M.
3. Draw the perpendicular bisector of MB and let it intersect AB in N. Thus, AN = 13 of AB = 6cm.

2. We know that, by triangle inequality property, construction of triangle is possible if sum of two sides of a triangle is greater than the third side. Here, AC + BC = 5cm which is less than AB (6cm) Thus, ΔABC is not possible.

   Also, by triangle inequality property, construction of triangle is possible, if difference of two sides of a triangle is less than the third side

   Here, AC – BC = 5cm, which is less than AB (6cm)

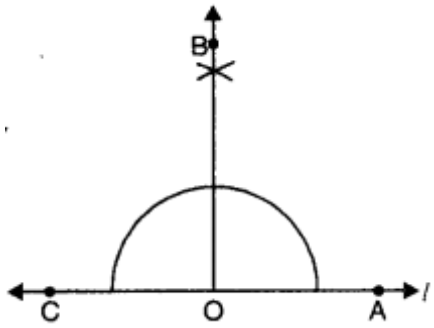
   Thus, ΔABC is possible.

3.

Steps of Construction :

1. Draw a ray OA.
2. With O as centre and any convenient radius, draw an arc, cutting OA at P.
3. With P as centre and same radius, draw an arc cutting the arc drawn in step 2 at Q.
4. With Q as centre and the same radius as in steps 2 and 3, draw an arc, cutting the arc drawn in step 2 at R.
5. With Q and R as centres and same radius, draw two arcs, cutting each other in S.
6. Join OS and produce to B. Thus, ∠AOB is the required angle of 90°

4.



Steps of Construction:

1. Draw any straight angle (say ∠AOC).
2. Bisect ∠AOC and join BO.
3. ∠AOB is the required bisector of straight angle AOC.

5.

Steps of Construction:

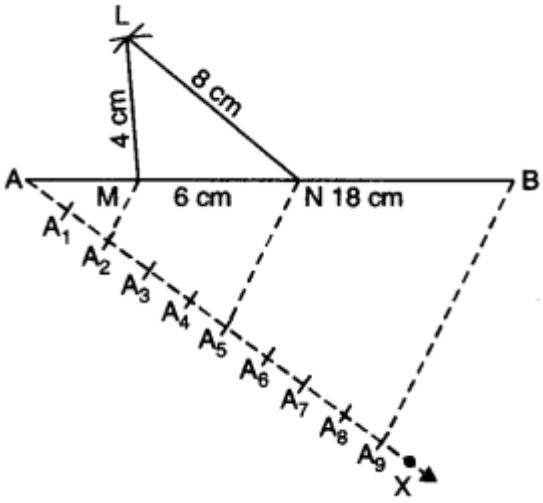
a. Let ∠AOB be any reflex angle.

b. With O as Centre and any convenient radius, draw an arc cutting OA in P and OB in Q.

c. With P and Q as centers, draw two arcs of radius little more than half of it and let they intersect each other in C. Join OC. Thus, OC is the required bisector. Angles so obtained are ∠AOC and ∠COB.

Short Answer:

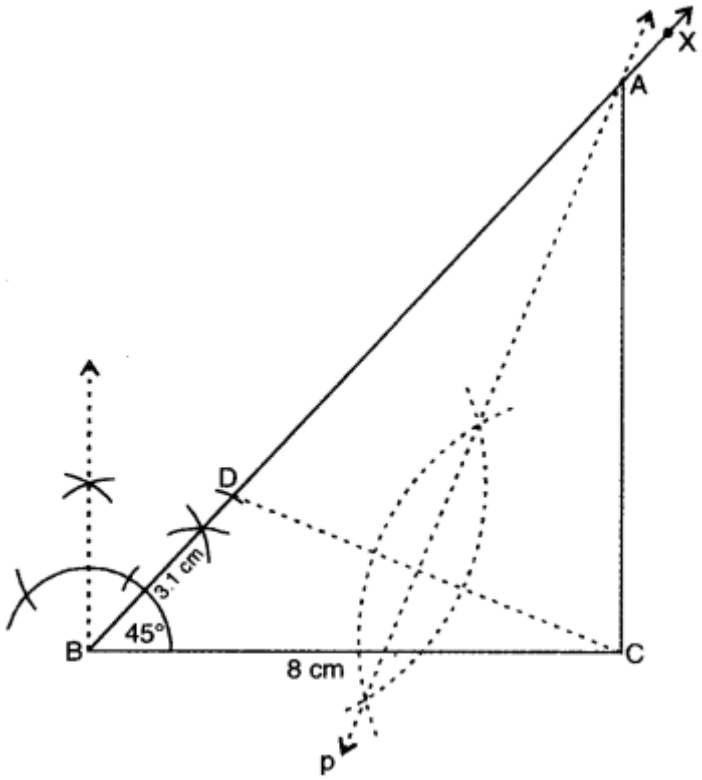
Ans: 1.



Steps of Construction:

1. Draw a line segment AB = 18cm.
2. At A, construct an acute angle ∠BAX (< 90°).
3. Mark 9 points on AX, such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7 = A_7A_8 = A_8A_9$.
4. Join A_9B.
5. From A_2 and A_5, draw $A_2M \parallel A_5N \parallel A_9B$, intersecting AB in M and N respectively.
6. With M as Centre and radius AM, draw an arc.
7. With N as Centre and radius NB, draw another arc intersecting the previous arc at L.
8. Join LM and LN. Thus, ΔLMN is the required triangle.

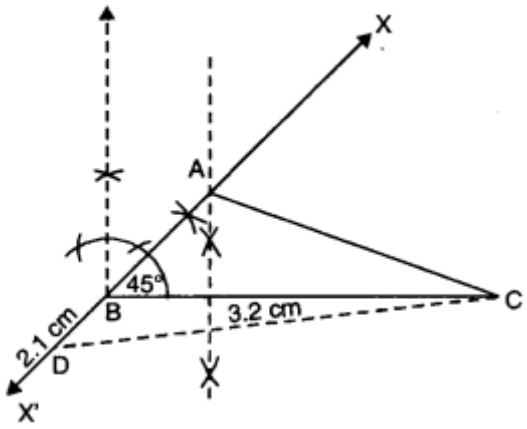
Ans: 2.



Steps of Construction:

1. Draw any line segment BC = 8cm.
2. At B, construct an angle ∠CBX = 45°.
3. From BX, cut off BD = 3.1cm.
4. Join DC.
5. Draw the perpendicular bisector ‘p’ of DC and let it intersect BX in A.
6. Join AC. Thus, ΔABC is the required triangle.

Ans: 3.

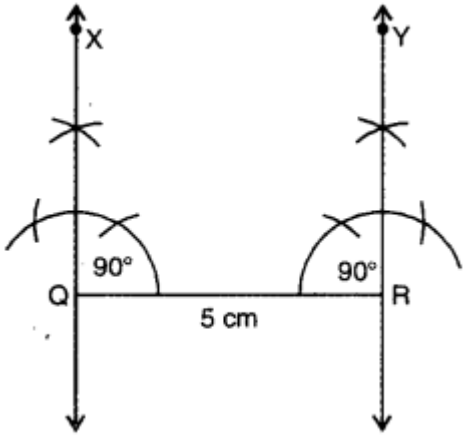


Steps of Construction:

1. Draw a line segment BC = 3.2cm.
2. At B, construct an angle ∠CBX = 45° and produce it to point X’.
3. Cut-off BD = 2.1cm and join CD.

4. Draw the perpendicular bisector of CD and let it intersect X'BX in A.
5. Join AC. Thus, ΔABC is the required triangle.

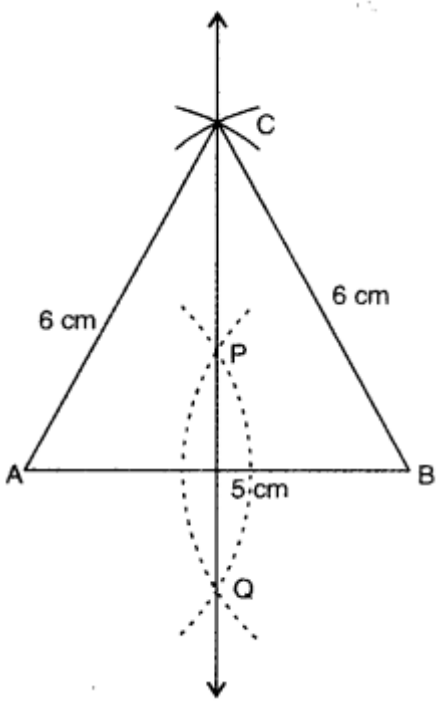
Ans: 4.



Steps of Construction:

1. Draw a line segment QR = 5cm.
2. With Q as Centre, construct an angle of 90° and let this line through Q is QX.
3. With R as Centre, construct an angle of 90° and let this line through R is RY. Yes, the perpendicular lines QX and RY are parallel.

Ans: 5.



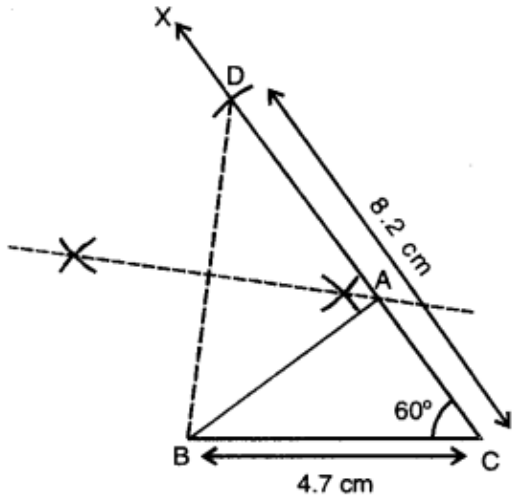
Steps of Construction:

1. Draw a line segment AB = 5cm.
2. With A and B as centers, draw two arcs of radius 6 cm and let they intersect each other in C.

3. Join AC and BC to get ΔABC.
4. With A and B as canters, draw two arcs of radius little more than half of AB. Let they intersect each other in P and Q. Join PQ and produce, to pass through C.

Long Answer:

Ans: 1



Given: In ΔABC, BC = 4.7cm, AB + AC = 8.2cm and ∠C = 60°.
Required: To construct ΔABC.
Steps of Construction:

1. Draw BC = 4.7cm.
2. Draw
3. From ray CX, cut off CD = 8.2cm.
4. Join BD.
5. Draw the perpendicular bisector of BD meeting CD at A.
6. Join AB to obtain the required triangle ABC.

Justification:
∵ A lies on the perpendicular bisector of BD, therefore, AB = AD
Now, CD = 8.2cm
⇒ AC + AD = 8.2cm
⇒ AC + AB = 8.2cm

Ans: 2.

5 cm

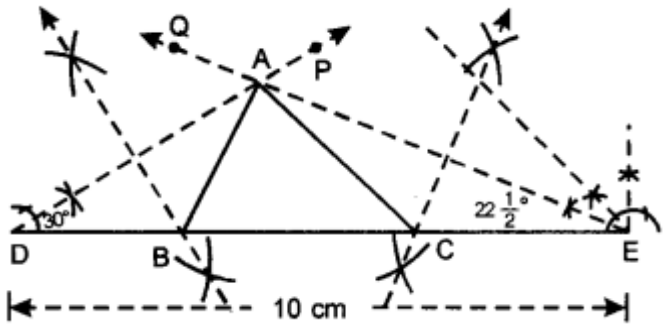
Here, perimeter of ΔXYZ = 14cm and one side XY = 5cm
∴ YZ + XZ = 14 - 5 = 9cm and ∠X = 45°.

Steps of Construction:

1. Draw a line segment XY = 5cm.
2. Construct an ∠YXA = 45° with the help of compass and ruler.

3. From ray XA, cut off XB = 9cm.
4. Join BY.
5. Draw perpendicular bisector of BY and let it intersect XB in Z.
6. Join ZY. Thus, ΔXYZ is the required triangle.

Ans: 3.



Given: In ΔABC,

AB + BC + CA = 10 cm, ∠B = 60° and ∠C = 45°.

Required: To construct ΔABC.

Steps of Construction:

1. Draw DE = 10cm.
2. At D, construct ∠EDP= 5 of 60°= 30° and at E, construct DEQ = 1 of 45o = 22°
3. Let DP and EQ meet at A.
4. Draw perpendicular bisector of AD to meet DE at B.
5. Draw perpendicular bisector of AE to meet DE at C.
6. Join AB and AC. Thus, ABC is the required triangle.

Ans: 4.

B S C

Steps of Construction:

1. Draw a line PQ and take any point S on it.
2. Construct the perpendicular SR on PQ.
3. From SR, cut a line segment SA = 6cm.
4. At the initial point A of the line segment AS, construct ∠SAB = 30° and ∠SAC = 30°.
5. The arms AB and AC of the angles ∠SAB and ∠SAC meet PQ in B and C respectively. Then, ΔABC is the required equilateral triangle with altitude of length 6cm.

Ans: 5.

Steps of Construction:

1. Draw a line segment PR = 8cm.
2. Draw the perpendicular bisector XY of the line segment PR. Let O be the point of intersection of PR and XY, so that O is the 8 cm mid-point of PR.
3. From OX, cut a line segment OS = 3cm and from OY, cut a line segment OQ = 3cm.
4. Join PS, SR, RQ and QP, then PQRS is the required rhombus.
5. Measure the length of segments PQ, QR, RS and SP, each is found to be 5cm long.

Assertion and Reason Answers-

1. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

2. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.