MATHEMATICS

Chapter 12: Heron's Formula

Heron's Formula

1. The region enclosed within a simple closed figure is called its **area**.
2. **Area of a triangle** $= \frac{1}{2} \times \text{base} \times \text{height}$
3. **Area of an equilateral triangle** $= \frac{\sqrt{3}}{4} a^2$sq units, where 'a' is the side length of an equilateral triangle.
4. **Semi-perimeter** is half of the perimeter.
5. If a, b and c denote the lengths of the sides of a triangle, then the area of the triangle is calculated by using **Heron's formula**, as given below:

   Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}, s = \text{ semi} - \text{perimeter} = \frac{a+b+c}{2}$
6. For every triangle, the values of (s – a), (s – b), and (s – c) are positive.
7. Area of a quadrilateral can be calculated by dividing the quadrilateral into two triangles and using Heron's formula for calculating area of each triangle.

Triangle

The plane closed figure, with three sides and three angles is called as a triangle.

Types of triangles:

Based on sides a) Equilateral b) Isosceles c) Scalene

Based on angles a) Acute angled triangle b) Right-angled triangle c) Obtuse angled triangle

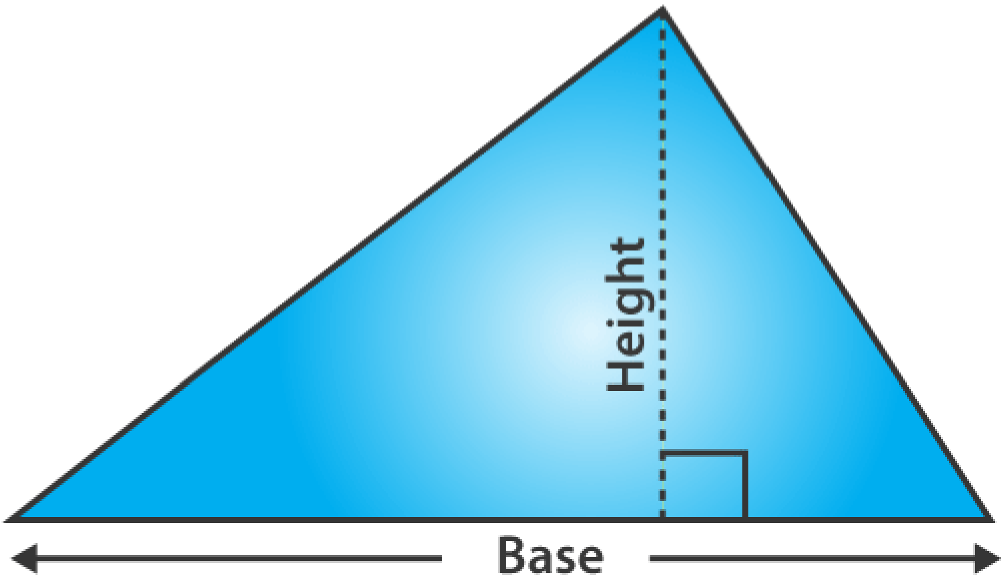
Area of a triangle

Area = (1/2) × base × height

In case of equilateral and isosceles triangles, if the lengths of the sides of triangles are given then, we use Pythagoras theorem in order to find the height of a triangle.

Area of a triangle is the region enclosed by it, in a two-dimensional plane. As we know, a triangle is a closed shape that has three sides and three vertices. Thus, the area of a triangle is the total space occupied within the three sides of a triangle. The general formula to find the area of the triangle is given by half of the product of its base and height.

In general, the term "area" is defined as the region occupied inside the boundary of a flat object or figure. The measurement is done in square units with the standard unit being square meters (m^2). For the computation of area, there are pre-defined formulas for squares, rectangles, circle, triangles, etc. In this article, we will learn the area of triangle formulas for different types of triangles, along with some example problems.



$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

Example: What is the area of a triangle with base b = 3 cm and height h = 4 cm?

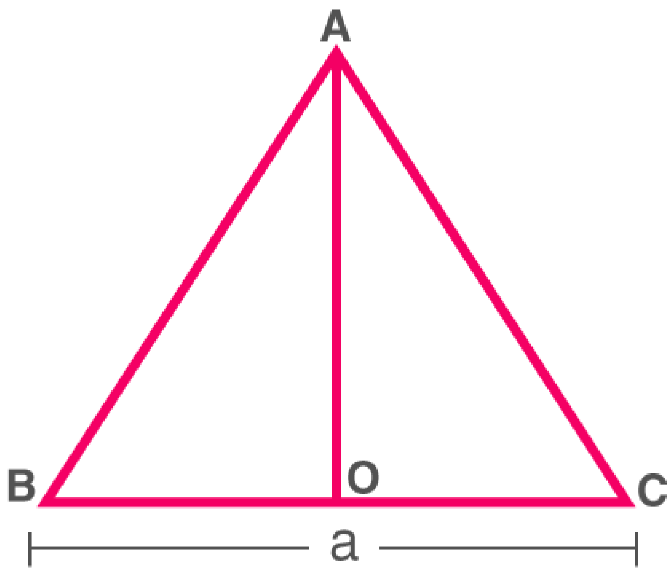
Using the formula,

Area of a Triangle, A = 1/2 × b × h = 1/2 × 4 cm × 3 cm = 2 cm × 3 cm = 6 cm^2

Apart from the above formula, we have Heron’s formula to calculate the triangle’s area, when we know the length of its three sides. Also, trigonometric functions are used to find the area when we know two sides and the angle formed between them in a triangle. We will calculate the area for all the conditions given here.

Area of an equilateral triangle

Consider an equilateral ΔABC, with each side as a unit. Let AO be the perpendicular bisector of BC. In order to derive the formula for the area of an equilateral triangle, we need to find height AO.



Using Pythagoras theorem,

$AC^2 = OA^2 + OC^2$

$OA^2 = AC^2 - OC^2$

Substitute AC = a, OC = a/2 in the above equation.

$OA^2 = a^2 - a^2/4$

$OA = \sqrt{3}a/2$

We know that the area of the triangle is:

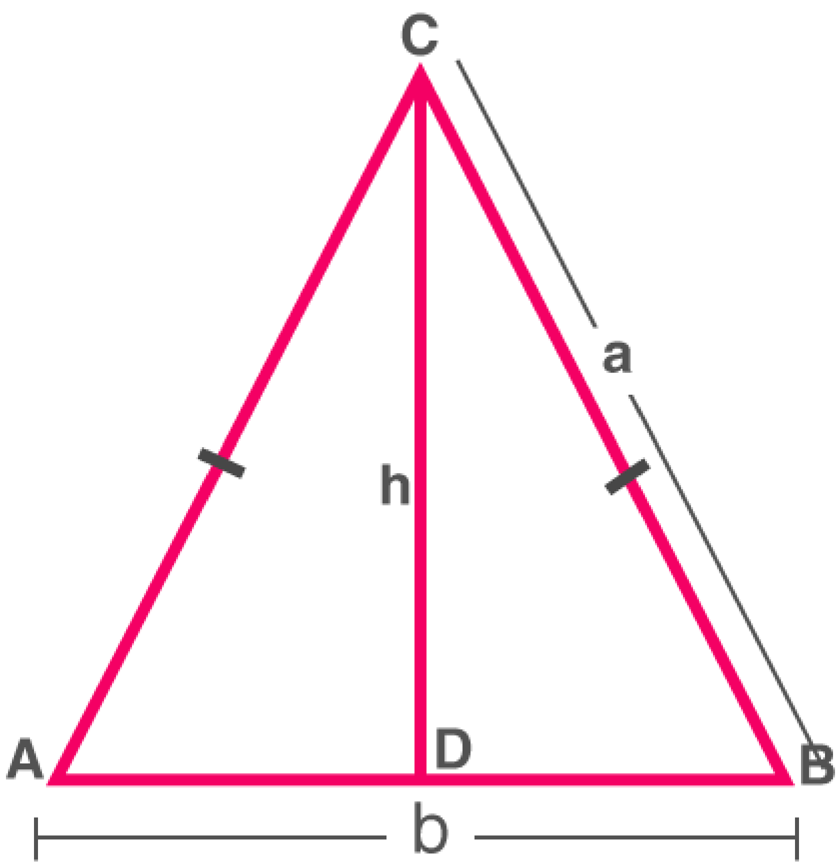
A = (1/2) × base × height

$A = (1/2) \times a \times (\sqrt{3}a/2)$

∴ Area of Equilateral triangle = $\sqrt{3}a^2/4$

Area of an isosceles triangle

Consider an isosceles ΔABC with equal sides as a units and base as b units.



Isosceles triangle ABC

The height of the triangle can be found by Pythagoras' Theorem:

$CD^2 = AC^2 - AD^2$

$\Rightarrow h^2 = a^2 - (b^2/4) = (4a^2 - b^2)/4$

$\Rightarrow h = (1/2)\sqrt{(4a^2 - b^2)}$

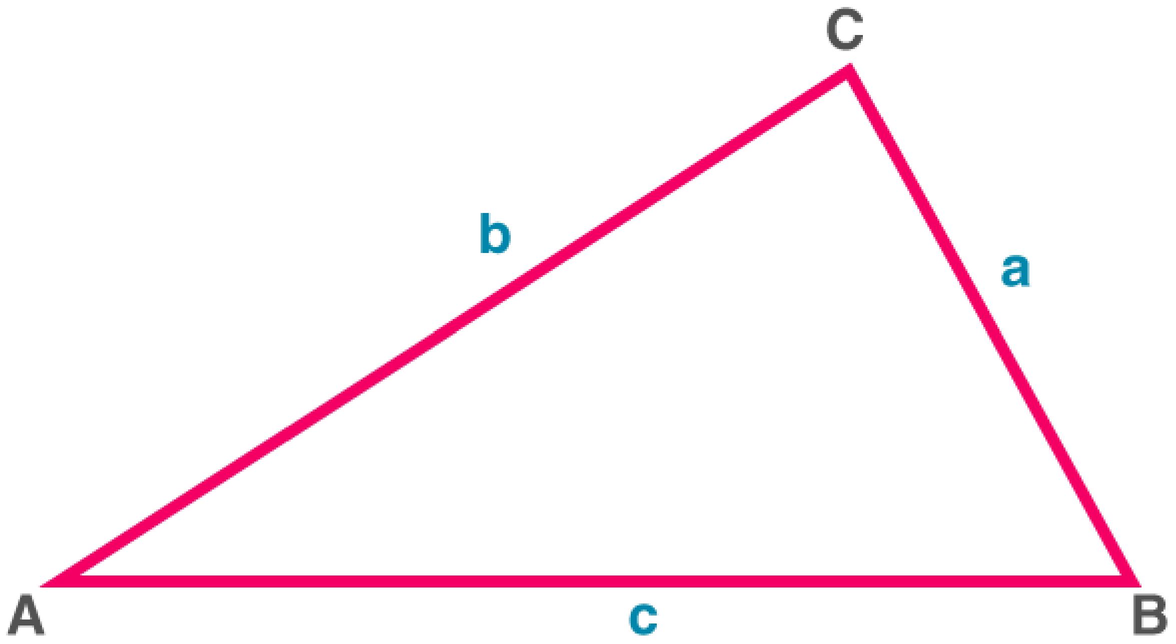
Area of triangle is A = (1/2) bh

$\therefore A = (1/2) \times b \times (1/2) \sqrt{(4a^2 - b^2)}$

$\therefore A = (1/4) \times b \times \sqrt{(4a^2 - b^2)}$

Area of a triangle – By Heron's formula

Area of a ΔABC, given sides a, b, c by Heron's formula (also known as Hero's Formula) is:



Triangle ABC

Find semi perimeter (s) = (a + b + c)/2

Area = $\sqrt{[s(s - a)(s - b)(s - c)]}$

This formula is helpful to find the area of a scalene triangle, given the lengths of all its sides.

Heron's Formula

Heron's formula is used to find the area of a triangle when we know the length of all its sides. It is also termed as Hero's Formula. We can Heron's formula to find different types of triangles, such as scalene, isosceles and equilateral triangles. We don't have to need to know the angle measurement of a triangle to calculate its area, by using Heron's formula.

Heron's formula is a formula to calculate the area of triangles, given the three sides of the triangle. This formula is also used to find the area of the quadrilateral, by dividing the quadrilateral into two triangles, along its diagonal.

If a, b and c are the three sides of a triangle, respectively, then Heron's formula is given by:

$$Area\ of\ triangle\ using\ three\ sides = \sqrt{s(s-a)(s-b)(s-c)}$$

Semiperimeter, s= Perimeter of triangle/2 = (a + b + c)/2

History of Heron's Formula

Hero of Alexandria was a great mathematician who derived the formula for the calculation of the area of a triangle using the length of all three sides. He also extended this idea to find the area of quadrilateral and also higher-order polygons. This formula has its huge applications in trigonometry such as proving the law of cosines or the law of cotangents, etc.

Proof of Heron's Formula

There are two methods by which we can derive Heron's formula.

- First, by using trigonometric identities and cosine rule.
- Secondly, solving algebraic expressions using the Pythagoras theorem.

Let us see one by one both the proofs or derivation.

Using Cosine Rule

Let us prove the result using the law of cosines:

Let a, b, c be the sides of the triangle and α, β, γ are opposite angles to the sides.

We know that the law of cosines is

$$Cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

Again, using trig identity, we have

$$Sin\gamma = \sqrt{1 - cos^2\gamma}$$

$$= \frac{\sqrt{4a^2b^2 - (a^2+b^2-c^2)^2}}{2ab}$$

Here, Base of triangle = a

Altitude = b sinγ

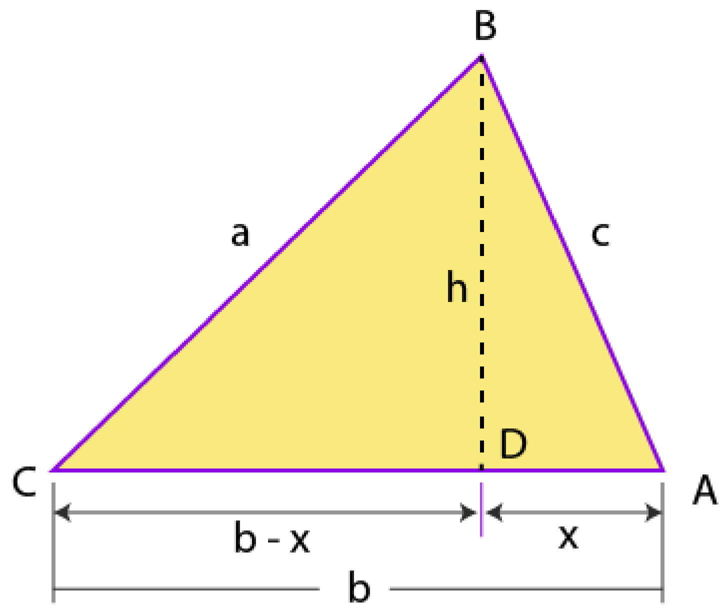
Now,

$$\begin{aligned}
A &= \frac{1}{2}(\text{base})(\text{altitude}) \\
&= \frac{1}{2}ab\sin\gamma \\
&= \frac{1}{4}\sqrt{4a^2b^2 - (a^2+b^2-c^2)^2} \\
&= \frac{1}{4}\sqrt{(2ab - (a^2+b^2-c^2))(2ab + (a^2+b^2-c^2))} \\
&= \frac{1}{4}\sqrt{(c^2 - (a-b)^2)((a+b)^2 - c^2)} \\
&= \sqrt{\frac{(c-(a-b))(c+(a-b))((a+b)-c)((a+b)+c)}{16}} \\
&= \sqrt{\frac{(b+c-a)}{2}\frac{(a+c-b)}{2}\frac{(a+b-c)}{2}\frac{(a+b+c)}{2}} \\
&= \sqrt{\frac{(a+b+c)}{2}\frac{(b+c-a)}{2}\frac{(a+c-b)}{2}\frac{(a+b-c)}{2}} \\
&= \sqrt{s(s-a)(s-b)(s-c)}.
\end{aligned}$$

Using Pythagoras Theorem

Area of a Triangle with 3 Sides

Area of ΔABC is given by



$A = 1/2\ bh$ _ _ _ _ (i)

Draw a perpendicular BD on AC

Consider a ΔADB

$x^2 + h^2 = c^2$

$x^2 = c^2 - h^2$—(ii)

$\Rightarrow x = \sqrt{(c^2 - h^2)}$——————(iii)

Consider a ΔCDB,

$(b - x)^2 + h^2 = a^2$

$(b-x)^2 = a^2 - h^2$

$b^2 - 2bx + x^2 = a^2 - h^2$

Substituting the value of x and x^2 from equation (ii) and (iii), we get

$b^2 - 2b\sqrt{(c^2 - h^2)} + c^2 - h^2 = a^2 - h^2$

$b^2 + c^2 - a^2 = 2b\sqrt{(c^2 - h^2)}$

Squaring on both sides, we get;

$(b^2 + c^2 - a^2)^2 = 4b^2(c^2 - h^2)$

$$\frac{(b^2+c^2-a^2)^2}{4b^2} = c^2 - h^2$$

$$h^2 = c^2 - \frac{(b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{4b^2c^2-(b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{(2bc)^2-(b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{[2bc+(b^2+c^2-a^2)][2bc-(b^2+c^2-a^2)]}{4b^2}$$

$$h^2 = \frac{[(b^2+2bc+c^2)-a^2][a^2-(b^2-2bc+c^2)]}{4b^2}$$

$$h^2 = \frac{[(b+c)^2-a^2].[a^2-(b-c)^2]}{4b^2}$$

$$h^2 = \frac{[(b+c)+a][(b+c)-a].[a+(b-c)][a-(b-c)]}{4b^2}$$

$$h^2 = \frac{(a+b+c)(b+c-a)(a+c-b)(a+b-c)}{4b^2}$$

The perimeter of a ΔABC is

P = a + b + c

$$\Rightarrow h^2 = \frac{P(P-2a)(P-2b)(P-2c)}{4b^2}$$

$$\Rightarrow h = \sqrt{P(P-2a)(P-2b)(P-2c)}2b$$

Substituting the value of h in equation (i), we get;

$$A = \frac{1}{2}b\frac{\sqrt{P(P-2a)(P-2b)(P-2c)}}{2b}$$

$$A = \frac{1}{4}\sqrt{(P(P-2a)(P-2b)(P-2c)}$$

$$A = \sqrt{\frac{1}{16}P(P-2a)(P-2b)(P-2c)}$$

$$A = \sqrt{\frac{P}{2}\left(\frac{P-2a}{2}\right)\left(\frac{P-2b}{2}\right)\left(\frac{P-2c}{2}\right)}$$

Semi perimeter(s) =

$$\frac{perimeter}{2} = \frac{P}{2}$$

$$\Rightarrow A = \sqrt{s(s-a)(s-b)(s-c)}$$

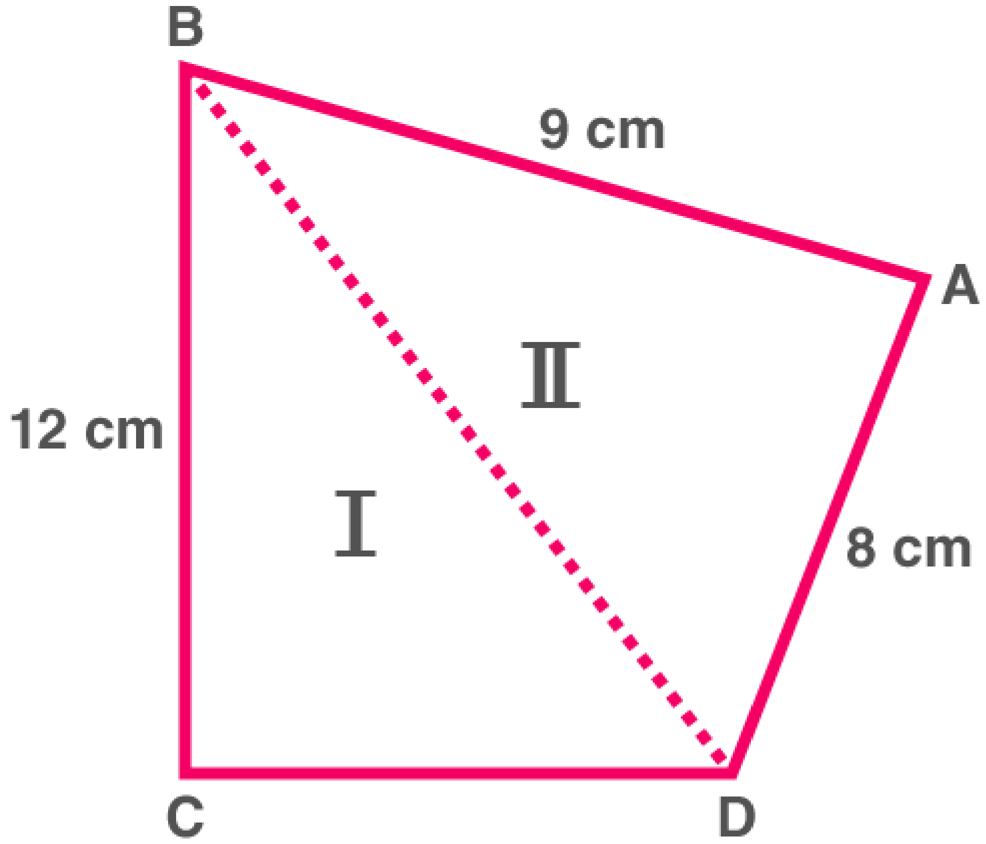
Note: Heron's formula is applicable to all types of triangles and the formula can also be derived using the law of cosines and the law of Cotangents.

Area of any polygon – By Heron's formula

For a quadrilateral, when one of its diagonal value and the sides are given, the area can be calculated by splitting the given quadrilateral into two triangles and use the Heron's formula.

Example: A park, in the shape of a quadrilateral ABCD, has $\angle C = 90^\circ$, AB = 9 cm, BC = 12 cm, CD = 5 cm and AD = 8 cm. How much area does it occupy?

⇒ We draw the figure according to the information given.



The figure can be split into 2 triangles ΔBCD and ΔABD

From ΔBCD, we can find BD (Using Pythagoras' Theorem)

$BD^2 = 12^2 + 5^2 = 169$

BD = 13cm

Semi-perimeter for ΔBCD $S_1 = (12 + 5 + 13)/2 = 15$

Semi-perimeter ΔABD $S_2 = (9 + 8 + 13)/2 = 15$

Using Heron's formula A_1 and A_2 will be:

$A_1 = \surd[15(15 - 12)(15 - 5)(15 - 13)]$

$A_1 = \surd(15 \times 3 \times 10 \times 2)$

$A_1 = \surd 900 = 30\ cm^2$

Similarly,

A_2 will be 35.49 cm^2.

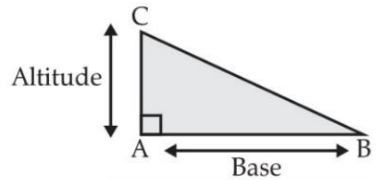
The area of the quadrilateral ABCD = $A_1 + A_2 = 65.49\ cm^2$

CHAPTER : 12 AREAS : HERON'S FORMULA

Heron's Formula

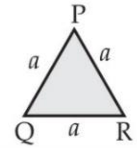
Area

Right-angled triangle



$\frac{1}{2} \times \text{Base} \times \text{Altitude}$

Equilateral triangle



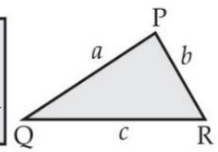
$\frac{\sqrt{3}}{4}\, a^2$; a = side of triangle

Heron's formula

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

where, a, b, c = sides of triangle

s = semi – perimeter = $\frac{a+b+c}{2}$



Applications

Find area of triangle of sides 122cm, 22cm, 120cm

By Heron's formula

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

where, $s = \frac{a+b+c}{2}$

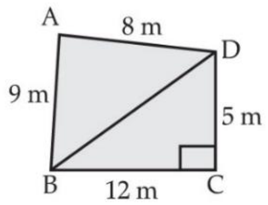
Here, $s = \frac{122+22+120}{2} = 132$ cm

$\therefore$ Area = $\sqrt{132\,(132\text{-}122)\,(132\text{-}22)\,(132\text{-}120)}$ cm^2

= $(\sqrt{132\,(10)\,(110)\,(12)})$cm^2

= 1320 cm^2

Area of quadrilateral ABCD with given dimensions :-



Area of ABCD = Area of $\triangle$ABD + area of $\triangle$BCD

Here, area of $\triangle$BCD = $\frac{1}{2} \times BC \times CD$

$= \frac{1}{2} \times 12 \times 5 = 30$ m^2

Here BD, = $\sqrt{BC^2+DC^2}$

BD = $\sqrt{12^2+5^2} = 13$ m

Area of ABD = $\sqrt{s\,(s\text{-}a)\,(s\text{-}b)\,(s\text{-}c)}$

where $s = \frac{a+b+c}{2} = \frac{(9+8+13)\text{ m}}{2} = 15$ m

Area = $\sqrt{15\,(15\text{-}9)\,(15\text{-}8)\,(15\text{-}13)}$ m^2

ABD = $\sqrt{15\,(6)\,(7)\,(2)}$ m^2

= 35.496 m^2

$\therefore$ Area of ABCD = (30+35.496) m^2

= 65.496 m^2

Important Questions

Multiple Choice questions-

Question 1. An isosceles right triangle has area $8cm^2$. The length of its hypotenuse is

(a) $\sqrt{32}$cm

(b) $\sqrt{16}$cm

(c) $\sqrt{48}$cm

(d) $\sqrt{24}$cm

Question 2. The perimeter of an equilateral triangle is 60m. The area is

(a) $10\sqrt{3}m^2$

(b) $15\sqrt{3}m^2$

(c) $20\sqrt{3}m^2$

(d) $100\sqrt{3}m^2$

Question 3. The sides of a triangle are 56cm, 60cm and 52cm long. Then the area of the triangle is

(a) $1322cm^2$

(b) $1311cm^2$

(c) $1344cm^2$

(d) $1392cm^2$

Question 4. The area of an equilateral triangle with side $2\sqrt{3}$cm is

(a) $5.196cm^2$

(b) $0.866cm^2$

(c) $3.496cm^2$

(d) $1.732cm^2$

Question 5. The length of each side of an equilateral triangle having an area of $9\sqrt{3}cm^2$ is

(a) 8cm

(b) 36cm

(c) 4cm

(d) 6cm

Question 6. If the area of an equilateral triangle is $16\sqrt{3}cm^2$, then the perimeter of the triangle is

(a) 48cm

(b) 24cm

(c) 12cm

(d) 36cm

Question 7. The sides of a triangle are 35cm, 54cm and 61cm. The length of its longest altitude is

(a) $16\sqrt{5}$cm

(b) $10\sqrt{5}$cm

(c) $24\sqrt{5}$cm

(d) 28cm

Question 8. The area of an isosceles triangle having base 2cm and the length of one of the equal sides 4 cm is

(a) $15\sqrt{}\mathrm{cm}^2$

(b) $\sqrt{\frac{15}{2}}\mathrm{cm}^2$

(c) $2\sqrt{15}\ 5\mathrm{cm}^2$

(d) $4\sqrt{15}\mathrm{cm}^2$

Question 9. The edges of a triangular board are 6 cm, 8cm and 10cm. The cost of painting it at the rate of 9 paise per cm^2 is

(a) Rs 2.00

(b) Rs 2.16

(c) Rs 2.48

(d) Rs 3.00

Question 10. The base of a right triangle is 48cm and its hypotenuse is 50cm. The area of the triangle is

(a) $168cm^2$

(b) $252cm^2$

(c) $336cm^2$

(d) $504cm^2$

Very Short:

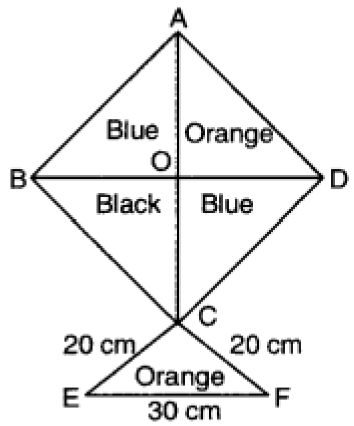
1. Find the area of an equilateral triangle having side 6cm.
2. If the perimeter of an equilateral triangle is 90m, then find its area.
3. If every side of a triangle is doubled, then find the percent increase in area of triangle so formed.
4. If the length of a median of an equilateral triangle is x cm, then find its area.

Short Questions:

1. Find the area of a triangle whose sides are 11m, 60m and 61m.
2. Suman has a piece of land, which is in the shape of a rhombus. She wants her two sons to work on the land and produce different crops. She divides the land in two equal parts by drawing a diagonal. If its perimeter is 400 m and one of the diagonals is of length 120 m, how much area each of them will get for his crops?
3. The perimeter of a triangular field is 144m and its sides are in the ratio 3 : 4 : 5. Find the length of the perpendicular from the opposite vertex to the side whose length is 60m.
4. Find the area of the triangle whose perimeter is 180 cm and two of its sides are of lengths 80 cm and 18 cm. Also, calculate the altitude of the triangle corresponding to the shortest side.

Long Questions:

1. Calculate the area of the shaded region.
2. The sides of a triangular park are 8m, 10m and 6m respectively. A small circular area of diameter 2m is to be left out and the remaining area is to be used for growing roses. How much area is used for growing roses? (Use n = 3.14)
3. OPQR is a rhombus, whose three vertices P, Q and R lie on the circle with Centre 0. If the radius of the circle is 12cm, find the area of the rhombus.
4. How much paper of each shade is needed to make a kite given in the figure, in which ABCD is a square with diagonal 60cm?



Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: the area of a triangle is 6 cm^2 whose sides are 3 cm, 4 cm and 5 cm respectively.

Reason: area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: the area of an equilateral triangle having each side 4 cm is $4\sqrt{3}$ cm^2

Reason: Area of an equilateral triangle = ($\sqrt{3}/4$) × a^2

Answer Key:

MCQ:

1. (a) $\sqrt{32}$cm
2. (d) $100\sqrt{3}m^2$
3. (c) $1344cm^2$
4. (a) $5.196cm^2$
5. (d) 6cm
6. (b) 24cm
7. (c) $24\sqrt{5}$cm
8. (a) $15\sqrt{}cm^2$
9. (b) Rs 2.16
10. (c) $336cm^2$

Very Short Answer:

1. Area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times 6 \times 6 = 9\sqrt{3}cm^2$

2.

$$\text{Side of an equilateral triangle} = \frac{\text{Perimeter}}{3} = \frac{90}{3} = 30 \text{ m}$$

$$\therefore \qquad \text{Its area} = \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times (30)^2$$

$$= \frac{\sqrt{3}}{4} \times 30 \times 30 = 225\sqrt{3} \text{ m}^2$$

3. Let the sides of the given triangle be, a units, b units and c units.

$$\therefore \qquad \text{Its area} = \sqrt{s(s-a)(s-b)(s-c)} \text{ sq. units}$$

Now, new sides of the triangle are $2a$ units, $2b$ units and $2c$ units.

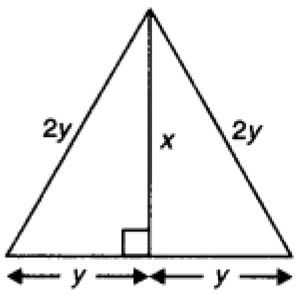
$$\text{Thus,} \qquad \text{its area} = \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$$

$$= 4\sqrt{s(s-a)(s-b)(s-c)} \text{ sq. units}$$

$$\text{Total increase in area} = 3\sqrt{s(s-a)(s-b)(s-c)} \text{ sq. units}$$

Hence, percent increase = 300%

4.



Let each equal sides of given equilateral triangle be $2y^2$. We know that median is also perpendicular bisector.

$\therefore y^2 + x^2 = 4y^2$

$\Rightarrow x^2 = 3y^2$

$\Rightarrow x = \sqrt{3}y$

or

$\Rightarrow y = \frac{x}{\sqrt{3}}$

Now, area of given triangle = $\frac{1}{2}$

$\times 2y \times X = y \times x = \frac{x}{\sqrt{3}} \times x = \frac{x2}{\sqrt{3}}$

Short Answer:

Ans: 1. Let a = 11m, b = 60m and c = 61m:

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{11+60+61}{2} = \frac{132}{2} = 66 \text{ m}$$

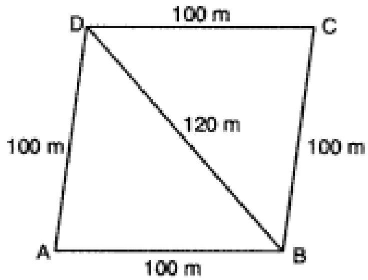
Now,
$$s-a = 66-11 = 55 \text{ m}$$
$$s-b = 66-60 = 6 \text{ m}$$
$$s-c = 66-61 = 5 \text{ m}$$

$$\therefore \text{Area of given triangle} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{66(55)(6)(5)}$$
$$= \sqrt{108900} = 330 \text{ sq. m.}$$

Ans: 2. Here, perimeter of the rhombus is 400m.

∴ Side of the rhombus = $\frac{400}{4}$ = 100m

Let diagonal BD = 120m and this diagonal divides the rhombus ABCD into two equal parts.



Now,
$$s = \frac{100+120+100}{2} = \frac{320}{2} = 160$$
$$\therefore \text{Area of } \Delta ABD = \sqrt{160(160-100)(160-100)(160-120)}$$
$$= \sqrt{160 \times 60 \times 60 \times 40} = 80 \times 60 = 4800 \text{ m}^2$$

Hence, area of land allotted to two sons for their crops is 4800m^2 each.

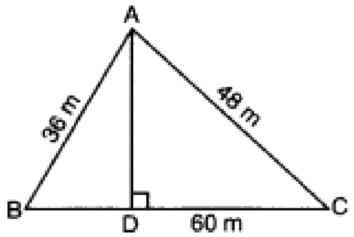
Ans: 3.

Let the sides of the triangle be 3x, 4x and 5x

∴ The perimeter of the triangular field = 144m

⇒ 3x + 4x + 5x = 144

⇒ 12x = 144



$$\Rightarrow \quad x = \frac{144}{12} = 12 \text{ m}$$

$\therefore$ Sides of the triangle are 3×12 m, 4×12 m, 5×12 m *i.e.*, 36 m, 48 m, 60 m

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{36+48+60}{2} = \frac{144}{2} = 72 \text{ m}$$

$$\text{Area of the } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{72(72-36)(72-48)(72-60)}$$

$$= \sqrt{72(36)(24)(12)} = \sqrt{746496} = 864 \text{ m}^2$$

$$\therefore \quad \text{Also, ar}(\Delta ABC) = \frac{1}{2} \times AD \times BC = \frac{1}{2} \times AD \times 60 = 30 \times AD$$

$$\therefore \quad 30 \times AD = 864$$

$$AD = \frac{864}{30} = 28.8 \text{ m}$$

Ans: 4. Perimeter of given triangle = 180cm

Two sides are 18cm and 80cm

∴ Third side = 180 - 18 - 80 = 82cm

$$s = \frac{180}{2} = 90 \text{ cm}$$

$$\text{Area of triangle} = \sqrt{90(90-18)(90-80)(90-82)}$$

$$= \sqrt{90 \times 72 \times 10 \times 8} = \sqrt{518400} = 720 \text{ cm}^2$$

$$\text{Also,} \quad \frac{1}{2} \times 18 \times h = 720$$

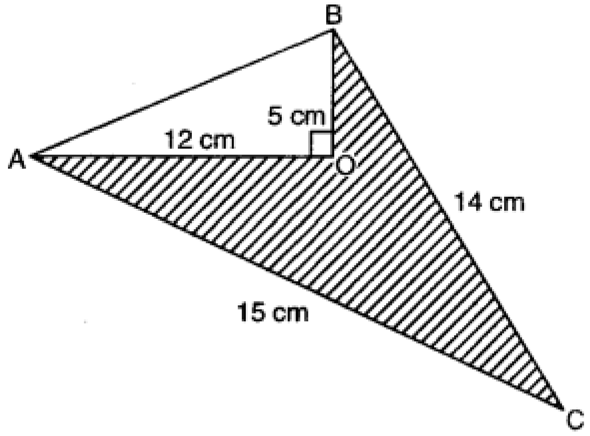
$$h = \frac{720}{9} = 80 \text{ cm}$$

Hence, area of triangle is 720cm^2 and altitude of the triangle corresponding to the shortest side is

80cm.

Long Answer:

Ans: 1



$$\text{Area of } \Delta AOB = \frac{1}{2} \times OA \times OB$$
$$= \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$$

Also, $AB^2 = OA^2 + OB^2$
$$= 12^2 + 5^2$$
$$= 144 + 25 = 169$$

$\Rightarrow$ $AB = \sqrt{169} = 13$ cm

Now, in ΔABC, we have

$a = BC = 14$ cm, $b = CA = 15$ cm, $c = AB = 13$ cm

$$s = \frac{a+b+c}{2} = \frac{14+15+13}{2} = \frac{42}{2} = 21 \text{ cm}$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{21(21-14)(21-15)(21-13)}$$
$$= \sqrt{21 \times 7 \times 6 \times 8}$$
$$= \sqrt{3 \times 7 \times 7 \times 2 \times 3 \times 2 \times 2 \times 2}$$

Ans: 2. The sides of the triangular park are 8m, 10m and 6m.

$$\therefore s = \frac{a+b+c}{2} = \frac{8+10+6}{2} = \frac{24}{2} = 12 \text{ m}$$

$$\text{Area of the park} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{12(12-8)(12-10)(12-6)}$$
$$= \sqrt{12 \times 4 \times 2 \times 6}$$
$$= \sqrt{2 \times 2 \times 3 \times 2 \times 2 \times 2 \times 2 \times 3}$$
$$= 2 \times 2 \times 2 \times 3 = 24 \text{ m}^2$$

Radius of the circle = $\frac{2}{2}$ = 1m

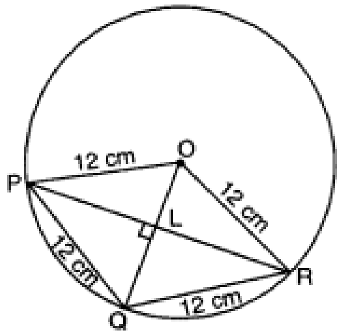
Area of the circle = $\pi r^2 = 3.14 \times 1 \times 1 = 3.14 m^2$

$\therefore$ Area to be used for growing roses = Area of the park - area of the circle

= 24 - 3.14 = 20.86m^2

Ans: 3. Since diagonals bisect each other at 90°.

∴ In right ΔQLR, $(LR)^2 + (LQ)^2 = (QR)^2$



$$\Rightarrow \left(\frac{PR}{2}\right)^2 + \left(\frac{OQ}{2}\right)^2 = (QR)^2$$

$$\Rightarrow \left(\frac{PR}{2}\right)^2 = (12)^2 - \left(\frac{12}{2}\right)^2 \quad [\because OQ = r = 12 \text{ cm}]$$

$$\Rightarrow \frac{PR^2}{4} = 144 - 36$$

$$\Rightarrow PR^2 = 4 \times 108 = 432$$

$$PR = \sqrt{144 \times 3} = 12\sqrt{3} \text{ cm}$$

$$\text{Area of rhombus OPQR} = \frac{1}{2} \times \text{product of diagonals}$$

$$= \frac{1}{2} \times OQ \times PR = \frac{1}{2} \times 12 \times 12\sqrt{3}$$

$$= 72\sqrt{3} \text{ cm}^2$$

Ans: 4. Since diagonals of a square are of equal length and bisect each other at right angles, therefore,

Area of $\Delta AOD = \frac{1}{2} \times 30 \times 30 = 450 cm^2$

Area of ΔAOD = Area of ΔDOC = Area of ΔBOC

= Area of $\Delta AOB = 450 cm^2$

[∵ $\Delta AOD = \Delta AOB \cong \Delta BOC \cong \Delta COD$,

∵ they^2 have equal area]

Now, area of ACEF (by Heron's formula)

Here a = 20cm, b = 20cm and c = 30cm

$$\Rightarrow \quad s = \frac{20+20+30}{2} = \frac{70}{2} = 35 \text{ cm}$$

$$\text{Area of } \Delta CEF = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{35(35-20)(35-20)(35-30)}$$

$$= \sqrt{35(15)(15)(5)} = 75\sqrt{7} \text{ cm}^2 \cong 198.4 \text{ cm}^2$$

Now, area of orange shaded paper in kite

= Area of ΔAOD + Area of ΔCEF

= $450 cm^2 + 198.4 cm^2$

= $648.4 cm^2$

Area of blue shaded paper in kite

= Area of ΔAOB + Area of ΔCOD

= $450cm^2 + 450cm^2 = 900cm^2$

Area of black shaded paper in kite = Area of ΔBOC = $450cm^2$.

Assertion and Reason Answers-

1. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

2. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.