MATHEMATICS

Chapter 6: Lines and Angles

Lines and Angles

Introduction to line and the terms related to it

- A **line** is a breadthless length which has no end point. Here, AB is a line and it is denoted by $\overleftrightarrow{AB}$.

- A **line segment** is a part of a line which has two end points. Here, AB is a line segment and it is denoted by $\overline{AB}$.

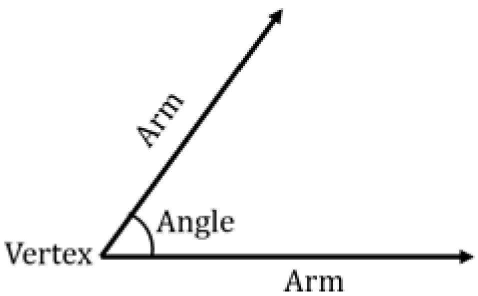
- A **ray** is a part of a line which has only one end point. Here, AB is a ray and it is denoted by $\overrightarrow{AB}$.

Collinear/Non-collinear points

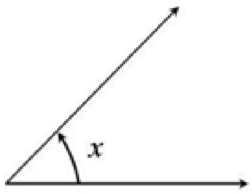
- Three or more points which lie on the same line are called **collinear points**.
- Three or more points which do not lie on a straight line are called **non-collinear points**.

Introduction to Angle

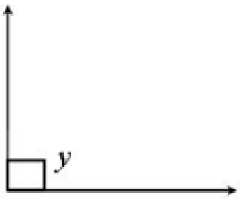
- An **angle** is formed when two rays originate from the same end point.
- The rays making an angle are called the **arms** of the angle.
- The end point from where the two rays originate to form an angle is called the **vertex** of the angle.



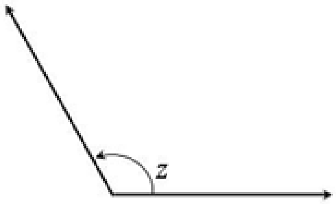
Types of angles:



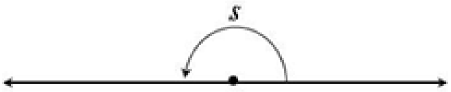
(i) Acute angle: $0^\circ < x < 90^\circ$



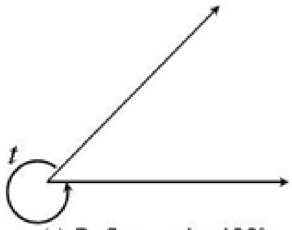
(ii) Right angle: $y = 90^\circ$



(iii) Obtuse angle: $90^\circ < z < 180^\circ$



(iv) Straight angle: $s = 180^\circ$

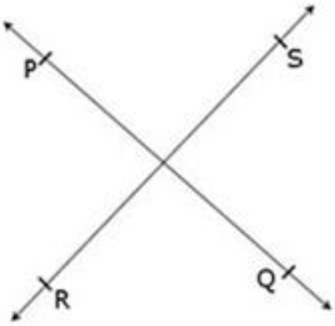


(v) Reflex angle: $180^\circ < t < 360^\circ$

Pair of Angles

- Two angles whose sum is 90° are called **complementary angles.**
- Two angles whose sum is 180° are called **supplementary angles.**

Intersecting and non-intersecting lines

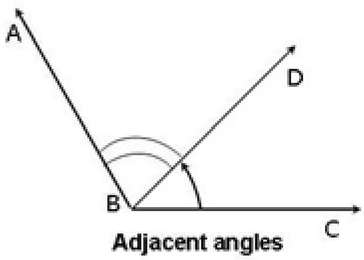


(i) Intersecting lines

(ii) Non–intersecting (parallel) lines

Adjacent angles

Two angles are **adjacent**, if they have a common vertex, a common arm and their non–common arms are on different sides of the common arm.



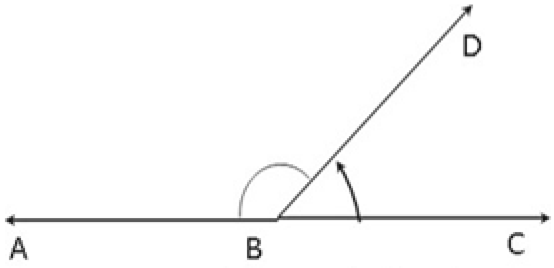
Adjacent angles

In the figure, $\angle ABD$ and $\angle DBC$ are adjacent angles.

Linear pair of angles

If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and vice-versa. This property is called as the **linear pair axiom** and the angles are called **linear pair of angles**.

In the figure, $\angle ABD$ and $\angle DBC$ are linear pair of angles i.e. $\angle ABD + \angle DBC = 180°$.

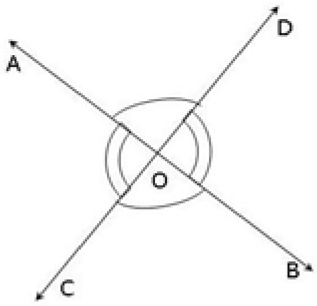


Linear pair of angles

If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line.

Vertically opposite angles

- The **vertically opposite angles** formed when two lines intersect each other.
- There are two pairs of vertically opposite angles in the given figure and they are $\angle AOD$ and $\angle BOC$, $\angle AOC$ and $\angle BOD$.

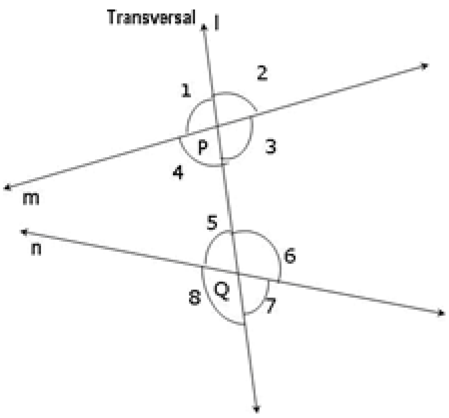


If two lines intersect each other, then the **vertically opposite angles are equal**.

Transversal

A line which intersects two or more lines at distinct points is called a **transversal**.

Pair of angles when a transversal intersects two lines



- **Corresponding angles:**

a) $\angle 1$ and $\angle 5$

b) $\angle 2$ and $\angle 6$

c) $\angle 4$ and $\angle 8$

d) $\angle 3$ and $\angle 7$

- **Alternate interior angles:**

  a) $\angle 4$ and $\angle 6$

  b) $\angle 3$ and $\angle 5$

- **Alternate exterior angles:**

  a) $\angle 1$ and $\angle 7$

  b) $\angle 2$ and $\angle 8$

- Interior angles on the same side of the transversal are referred as co-interior angles/ allied angles/ consecutive interior angles and they are:

  a) $\angle 4$ and $\angle 5$

  b) $\angle 3$ and $\angle 6$

If a transversal intersects two parallel lines, then

- Each pair of **corresponding angles are equal**.
- Each pair of **alternate interior angles are equal**.
- Each pair of interior angles on the same side of the transversal are supplementary.

If a transversal intersects two lines

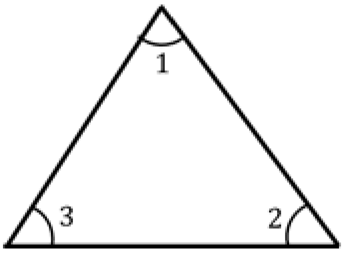
- Such that a pair of **corresponding angles** is equal, then the two **lines are parallel**.
- Such that a pair of **alternate interior angles** is equal, then the two **lines are parallel**.
- Such that a pair of **interior angles** on the same side of the transversal is supplementary, then the two **lines are parallel**.
- Such that the bisectors of a pair of **corresponding angles** are parallel, then the two **lines are parallel**.

Lines parallel to the same line

Two lines which are parallel to the same line are parallel to each other. This holds for more than two lines also i.e. if two or more lines are parallel to the same line then they will be parallel to each other.

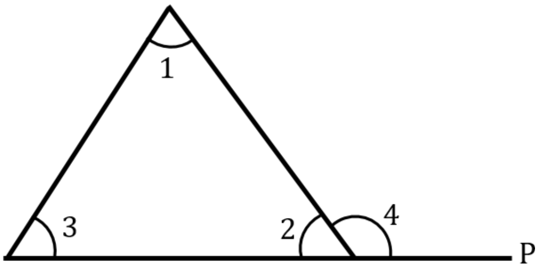
Angle sum property of a triangle

- The sum of the angles of a triangle is 180°. This is known as the **angle sum property of a triangle**.



Here, $\angle 1 + \angle 2 + \angle 3 = 180°$.

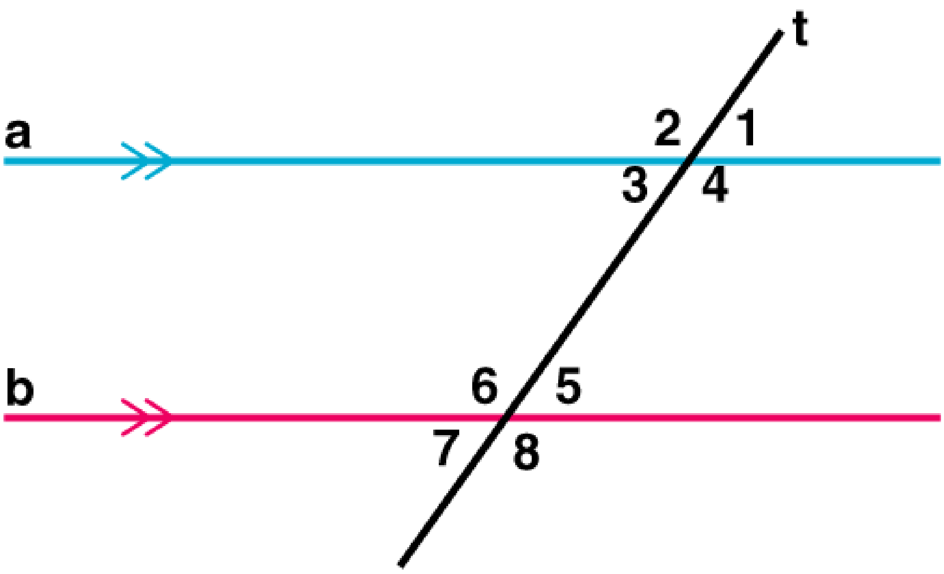
- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles. This is known as the **exterior angle property of a triangle**.



Here, $\angle 4 = \angle 1 + \angle 3$.

- An exterior angle of a triangle is greater than either of its interior opposite angles. In the above figure, $\angle 4 > \angle 1$ and $\angle 4 > \angle 3$.

Parallel lines with a transversal



- ∠1 = ∠5, ∠2 = ∠6, ∠4 = ∠8 and ∠3 = ∠7 (Corresponding angles)
- ∠3 = ∠5, ∠4 = ∠6 (Alternate interior angles)
- ∠1 = ∠7, ∠2 = ∠8 (Alternate exterior angles)

Angles and types of angles

When 2 rays originate from the same point at different directions, they form an angle.

The rays are called arms and the common point is called the vertex

Types of angles:

- Acute angle $0^\circ < a < 90^\circ$
- Right angle $a = 90^\circ$
- Obtuse angle: $90^\circ < a < 180^\circ$
- Straight angle $= 180^\circ$
- Reflex Angle $180^\circ < a < 360^\circ$
- Angles that add up to 90° are complementary angles
- Angles that add up to 180° are called supplementary angles.

Intersecting Lines and Associated Angles

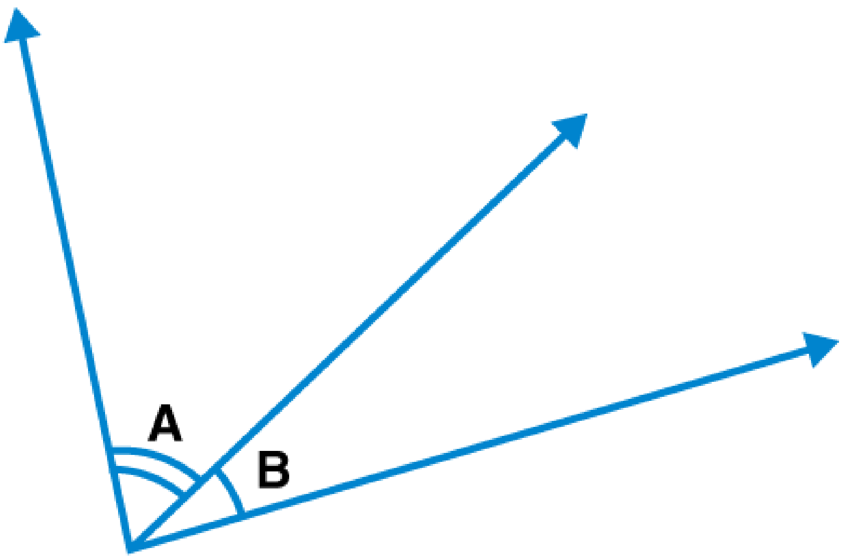
Intersecting and Non-Intersecting lines

When 2 lines meet at a point they are called intersecting

When 2 lines never meet at a point, they are called non-intersecting or parallel lines

Adjacent angles

2 angles are adjacent if they have the same vertex and one common point.



Linear Pair

When 2 adjacent angles are supplementary, i.e they form a straight line (add up to 180∘), they are called a linear pair.

Vertically opposite angles

When two lines intersect at a point, they form equal angles that are vertically opposite to each other.

Basic Properties of a Triangle

All the properties of a triangle are based on its sides and angles. By the definition of triangle, we know that it is a closed polygon that consists of three sides and three vertices. Also, the sum of all three internal angles of a triangle equal to 180°.

Depending upon the length of sides and measure of angles, the triangles are classified into different types of triangles.

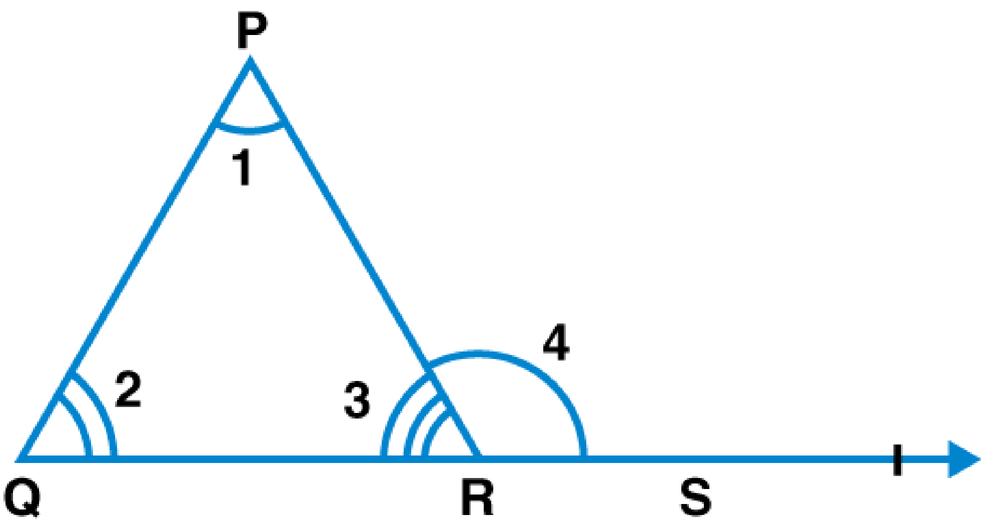
In the beginning, we start from understanding the shape of triangles, its types and properties, theorems based on it such as Pythagoras theorem, etc. In higher classes, we deal with trigonometry, where the right-angled triangle is the base of the concept. Let us learn here some of the fundamentals of the triangle by knowing its properties.

Triangle and sum of its internal angles

Sum of all angles of a triangle add up to 180∘

An exterior angle of a triangle = sum of opposite internal angles

– If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles



– $\angle 4 = \angle 1 + \angle 2$

Types of Triangle

Based on the Sides	Based on the Angles
Scalene Triangle	Acute angled Triangle
Isosceles Triangle	Right angle Triangle
Equilateral Triangle	Obtuse-angled Triangle

So before, discussing the properties of triangles, let us discuss types of triangles given above.

Scalene Triangle: All the sides and angles are unequal.

Isosceles Triangle: It has two equal sides. Also, the angles opposite these equal sides are equal.

Equilateral Triangle: All the sides are equal and all the three angles equal to 60°.

Acute Angled Triangle: A triangle having all its angles less than 90°.

Right Angled Triangle: A triangle having one of the three angles exactly 90°.

Obtuse Angled Triangle: A triangle having one of the three angles more than 90°.

Triangle Formula

- Area of a triangle is the region occupied by a triangle in a two-dimensional plane. The dimension of the area is square units. The formula for area is given by;

  Area = 1/2 x Base x Height

- The perimeter of a triangle is the length of the outer boundary of a triangle. To find the perimeter of a triangle we need to add the length of the sides of the triangle.

  $P = a + b + c$

- Semi-perimeter of a triangle is half of the perimeter of the triangle. It is represented by s.

  $s = (a + b + c)/2$

  where a, b and c are the sides of the triangle.

- By Heron's formula, the area of the triangle is given by:

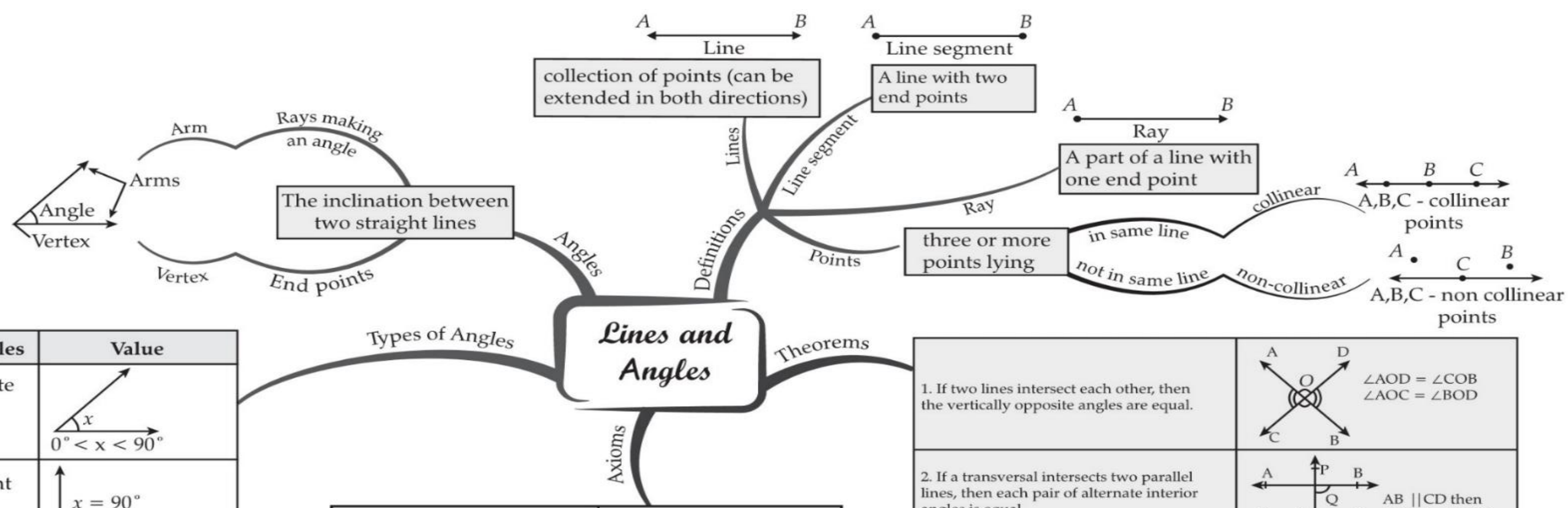
  $A = \sqrt{[s(s - a)(s - b)(s - c)]}$

  where 's' is the semi-perimeter of the triangle.

- By the Pythagorean theorem, the hypotenuse of a right-angled triangle can be calculated by the formula:

  $\text{Hypotenuse}^2 = \text{Base}^2 + \text{Perpendicular}^2$

CHAPTER : 6 LINES AND ANGLES



Angles	Value
Acute	$0^\circ < x < 90^\circ$
Right	$x = 90^\circ$
Obtuse	$90^\circ < x < 180^\circ$
Straight line	$x = 180^\circ$
Reflex	$180^\circ < x < 360^\circ$

Axioms

1. If a ray stands on a line, then the sum of two adjacent angles so formed is 180°	AB is a line then, $\angle AOC + \angle BOC = 180^\circ$
2. If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line.	If $\angle AOC + \angle BOC = 180^\circ$ then AB is a straight line
3. If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.	Here PQ || RS then $\angle ABQ = \angle BCS$
4. If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.	If $\angle BQR = \angle QRC$ and $\angle AQP = \angle QRD$ then, AB || CD

Theorems

1. If two lines intersect each other, then the vertically opposite angles are equal.	$\angle AOD = \angle COB$ $\angle AOC = \angle BOD$
2. If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal	AB ||CD then $\angle BQR = \angle CRQ$
3. If transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel	if $\angle BQR = \angle CRQ$ then AB ||CD
4. If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.	AB ||CD then, $\angle AQR + \angle CRQ = 180^\circ$
5. If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.	if $\angle AQR + \angle CRQ = 180^\circ$ then AB ||CD
6. Lines which are parallel to the same line are parallel to each other.	AB ||CD and CD ||EF then AB ||EF
7. The sum of all interior angles of a triangle is 180°	$\angle A + \angle B + \angle C = 180^\circ$

Important Questions

Multiple Choice Questions-

Question 1. In a right-angled triangle where angle A = 90° and AB = AC. What are the values of angle B?

(a) 45°

(b) 35°

(c) 75°

(d) 65°

Question 2. In a triangle ABC if ∠A = 53° and ∠C = 44° then the value of ∠B is:

(a) 46°

(b) 83°

(c) 93°

(d) 73°

Question 3. Given four points such that no three of them are collinear, then the number of lines that can be drawn through them are:

(a) 4 lines

(b) 8 lines

(c) 6 lines

(d) 2 lines

Question 4. If one angle of triangle is equal to the sum of the other two angles then triangle is:

(a) Acute triangle

(b) Obtuse triangle

(c) Right triangle

(d) None of these

Question 5. How many degrees are there in an angle which equals one-fifth of its supplement?

(a) 15°

(b) 30°

(c) 75°

(d) 150°

Question 6. Sum of the measure of an angle and its vertically opposite angle is always.

(a) Zero

(b) Thrice the measure of the original angle

(c) Double the measure of the original angle

(d) Equal to the measure of the original angle

Question 7. If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

(a) Equal

(b) Complementary

(c) Supplementary

(d) corresponding

Question 8. The bisectors of the base angles of an isosceles triangle ABC, with AB = AC, meet at O. If ∠B = ∠C = 50Â°. What is the measure of angle O?

(a) 120°

(b) 130°

(c) 80°

(d) 150°

Question 9. The angles of a triangle are in the ratio 2 : 3 : 4. The angles, in order, are :

(a) 80°, 40°, 60°

(b) 20°, 60°, 80°

(c) 40°, 60°, 80°

(d) 60°, 40°, 80°

Question 10. An acute angle is:

(a) More than 90 degrees

(b) Less than 90 degrees

(c) Equal to 90 degrees

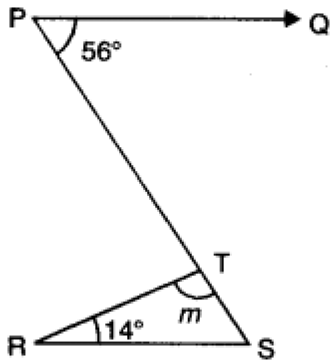
(d) Equal to 180 degrees

Very Short:

1. If an angle is half of its complementary angle, then find its degree measure.

2. The two complementary angles are in the ratio 1 : 5. Find the measures of the angles.

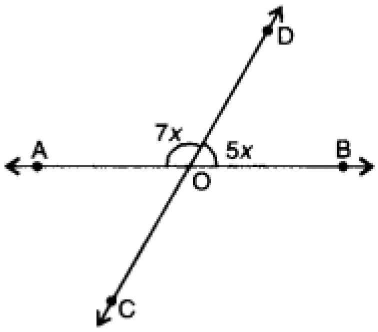
3. In the given figure, if PQ || RS, then find the measure of angle m.



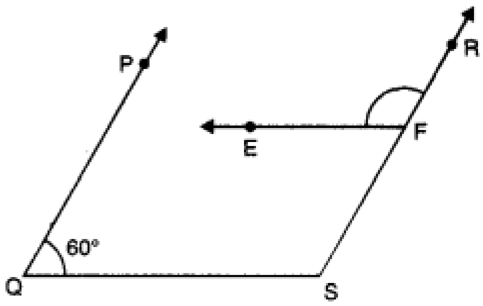
4. If an angle is 14o more than its complement, then find its measure.

5. If AB || EF and EF || CD, then find the value of x.

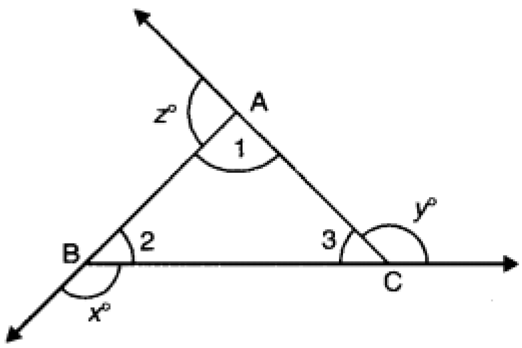
6. In the given figure, lines AB and CD intersect at O. Find the value of x.



7. In the given figure, PQ || RS and EF || QS. If ∠PQS = 60°, then find the measure of ∠RFE.

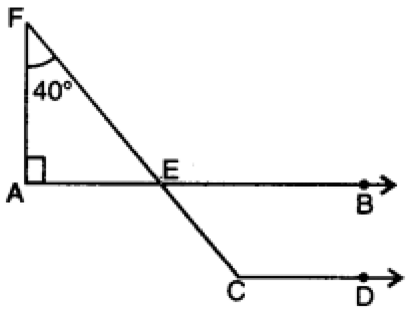


8. In the given figure, if x°, y° and z° are exterior angles of ΔABC, then find the value of x° + y° + z°.

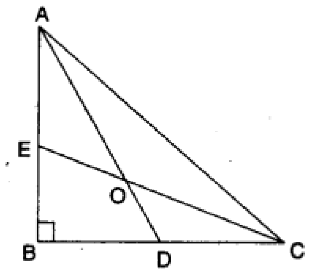


Short Questions:

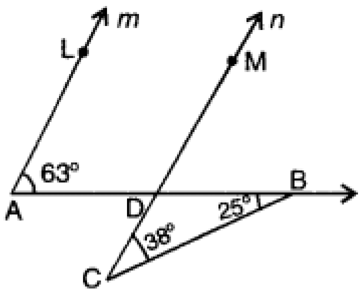
1. In the given figure, AB || CD, ∠FAE = 90°, ∠AFE = 40°, find ∠ECD.



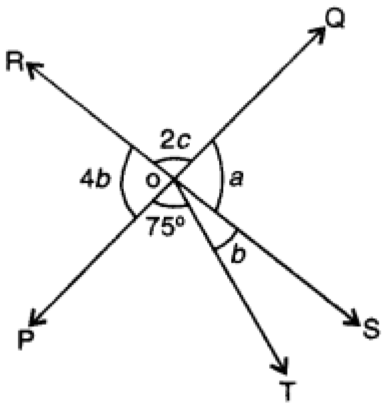
2. In the fig., AD and CE are the angle bisectors of ∠A and ∠C respectively. If ∠ABC = 90°, then find ∠AOC.



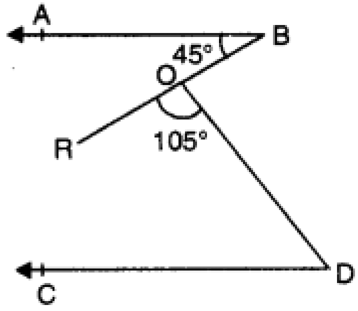
3. In the given figure, prove that m || n.



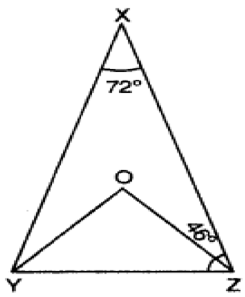
4. In the given figure, two straight lines PQ and RS intersect each other at O. If ∠POT = 75°, find the values of a, b, c.



5. In figure, if AB || CD. If ∠ABR = 45° and ∠ROD = 105°, then find ∠ODC.

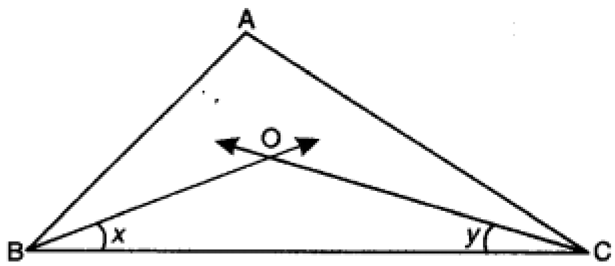


6. In the figure, ∠X = 72°, ∠XZY = 46°. If YO and ZO are bisectors of ∠XYZ and ∠XZY respectively of ΔXYZ, find ∠OYZ and ∠YOZ.

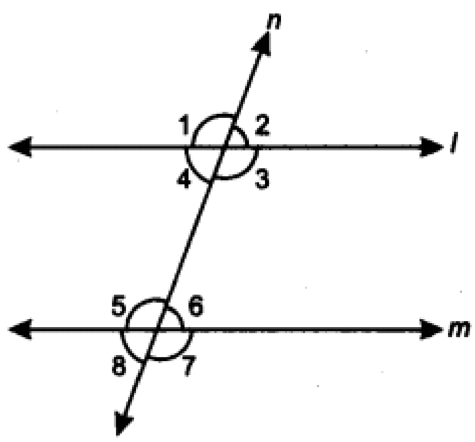


Long Questions:

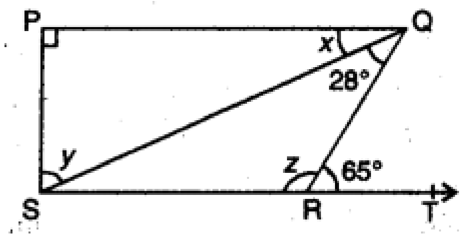
1. If two parallel lines are intersected by a transversal, prove that the bisectors of two pairs of interior angles form a rectangle.
2. If in ΔABC, the bisectors of ∠B and ∠C intersect each other at O. Prove that $\angle BOC = 90° \frac{1}{2} < A$



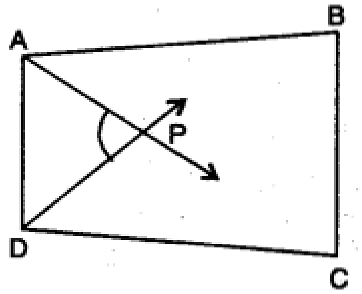
3. In figure, if l || m and ∠1 = (2x + y)°, ∠4 = (x + 2y)° and ∠6 = (3y + 20)°. Find ∠7 and ∠8.



4. In the given figure, if PQ ⊥ PS, PQ || SR, ∠SQR = 28o and ∠QRT = 65°. Find the values of x, y and z.



5. In figure, AP and DP are bisectors of two adjacent angles A and D of a quadrilateral ABCD. Prove that $2\angle APD = \angle B + \angle C$.



Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: Two adjacent angles always form a linear pair..

Reason: In a linear pair of angles two non-common arms are opposite rays.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

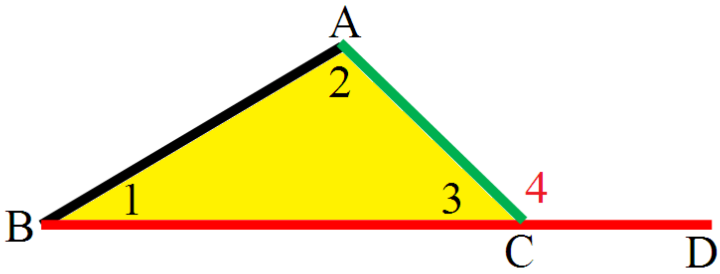
d) Assertion is wrong statement but reason is correct statement.

Assertion: A triangle can have tow obtuse angles.

Reason: Sum of the three angles in a triangle is always 180°.

Case Study Questions-

1. Read the Source/ Text given below and answer these questions:



Ashok is studying in 9th class in Govt School, Chhatarpur. Once he was at his home and was doing his geometry homework. He was trying to measure three angles of a triangle using the Dee, but his dee was old and his Dee's numbers were erased and the lines on the dee were visible. Let us help Ashok to find the angles of the triangle. He found that the second angle of the triangle was three times as large as the first. The measure of the third angle is double of the first angle.

Now answer the following questions:

i. What was the value of the first angle?

a. 30°
b. 45°
c. 60°
d. 90°

ii. What was the value of the third angle?

a. 30°
b. 45°
c. 60°
d. 90°

iii. What was the value of the second angle?

a. 30°
b. 45°
c. 60°
d. 90°

iv. What was the value of ∠4∠4 as shown the figure?

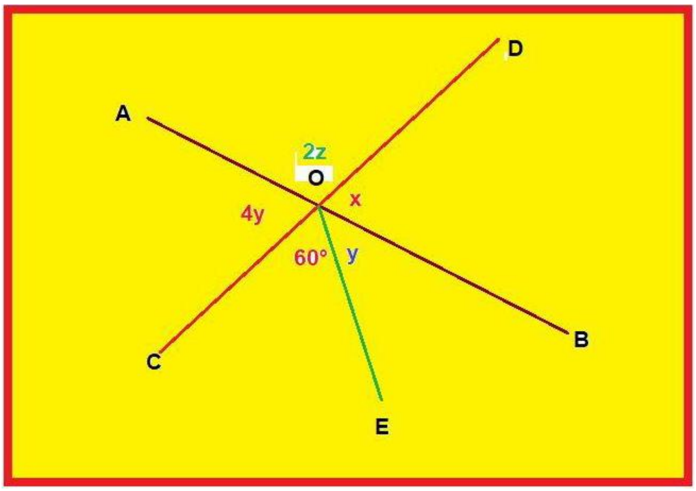
a. 120°
b. 45°
c. 60°
d. 90°

v. What was the sum of all three angles measured by Ashok using Dee?

a. 270°
b. 180°
c. 100°
d. 90°

2. Read the Source/ Text given below and answer any four questions:

Maths teacher draws a straight line AB shown on the blackboard as per the following figure.



i. Now he told Raju to draw another line CD as in the figure.
ii. The teacher told Ajay to mark ∠AOD as 2z.
iii. Suraj was told to mark ∠AOC as 4y.
iv. Clive Made and angle ∠COE=60o.
v. Peter marked ∠BOE and ∠BOD as y and x respectively.

Now answer the following questions:

i. What is the value of x?
   a. 48°
   b. 96°
   c. 100°
   d. 120°

ii. What is the value of y?
   a. 48°
   b. 96°
   c. 100°
   d. 24°

iii. What is the value of z?
   a. 48°
   b. 96°
   c. 42°
   d. 120°

iv. What should be the value of x + 2z?
   a. 148°
   b. 360°
   c. 180°
   d. 120°

v. What is the relation between y and z?

a. $2y + z = 90°$
b. $2y + z = 180°$
c. $4y + 2z = 120°$
d. $y = 2z$

Answer Key:

MCQ:

1. (a) 45°
2. (b) 83°
3. (c) 6 lines
4. (c) Right triangle
5. (b) 30°
6. (c) Double the measure of the original angle
7. (d) Corresponding
8. (b) 130°
9. (c) 40°, 60°, 80°
10. (b) Less than 90 degrees

Very Short Answer:

1. Let the required angle be x

∴ Its complement = $90° - x$

Now, according to given statement, we obtain

$x = \frac{1}{2}(90° - x)$

$\Rightarrow 2x = 90° - x$

$\Rightarrow 3x = 90°$

$\Rightarrow x = 30°$

Hence, the required angle is 30°.

2. Let the two complementary angles be x and 5x.

∴ $x + 5x = 90°$

$\Rightarrow 6x = 90°$

$\Rightarrow x = 15°$

3. Here, PQ || RS, PS is a transversal.

$\Rightarrow \angle PSR = \angle SPQ = 56°$

Also, $\angle TRS + m + \angle TSR = 180°$

$14° + m + 56° = 180°$

$\Rightarrow m = 180° – 14 – 56 = 110°$

4. Let the required angle be x

$\therefore$ Its complement = $90° – x$

Now, according to given statement, we obtain

$x = 90° – x + 14°$

$\Rightarrow 2x = 104°$

$\Rightarrow x = 52°$

Hence, the required angle is 52o.

5. Since EF || CD $\therefore$ $y + 150° = 180°$

$\Rightarrow y = 180° – 150° = 30°$

Now, $\angle BCD = \angle ABC$

$x + y = 70°$

$x + 30 = 70$

$\Rightarrow x = 70° – 30° = 40°$

Hence, the value of x is 40°

6. Here, lines AB and CD intersect at O.

$\therefore$ $\angle AOD$ and $\angle BOD$ forming a linear pair

$\Rightarrow \angle AOD + \angle BOD = 180°$

$\Rightarrow 7x + 5x = 180°$

$\Rightarrow 12x = 180°$

$\Rightarrow x = 15°$

7. Since PQ || RS

$\therefore \angle PQS + \angle QSR = 180°$

$\Rightarrow 60° + \angle QSR = 180°$

$\Rightarrow \angle QSR = 120°$

Now, EF || QS

$\Rightarrow \angle RFE = \angle QSR$ [corresponding $\angle$s]

$\Rightarrow \angle RFE = 120°$

8. We know that an exterior angle of a triangle is equal to sum of two opposite interior angles.

$\Rightarrow x° = \angle 1 + \angle 3$

$\Rightarrow y° = \angle 2 + \angle 1$

$\Rightarrow z° = \angle 3 + \angle 2$

Adding all these, we have

$x° + y° + z° = 2(\angle 1 + \angle 2 + \angle 3)$

$= 2 \times 180°$

$= 360°$

Short Answer:

Ans: 1. In AFAE,

ext. $\angle FEB = \angle A + F$

$= 90° + 40° = 130°$

Since AB || CD

$\therefore \angle ECD = FEB = 130°$

Hence, $\angle ECD = 130°$.

Ans: 2. $\because$ AD and CE are the bisector of $\angle A$ and $\angle C$

$$\therefore \quad \angle OAC = \frac{1}{2}\angle A \text{ and}$$

$$\angle OCA = \frac{1}{2}\angle C$$

$$\Rightarrow \quad \angle OAC + \angle OCA = \frac{1}{2}(\angle A + \angle C)$$

$$= \frac{1}{2}(180° - \angle B) \quad [\because \angle A + \angle B + \angle C = 180°]$$

$$= \frac{1}{2}(180° - 90°] \quad [\because \angle ABC = 90°]$$

$$= \frac{1}{2} \times 90° = 45°$$

In ΔAOC,

$\angle AOC + \angle OAC + \angle OCA = 180°$

$\Rightarrow \angle AOC + 45o = 180°$

$\Rightarrow \angle AOC = 180° - 45° = 135°$

Ans: 3. In ΔBCD,

ext. $\angle BDM = \angle C + \angle B$

$= 38° + 25° = 63°$

Now, $\angle LAD = \angle MDB = 63°$

But these are corresponding angles.

Hence, m | | n

Ans: 4. Here, 4b + 75° + b = 180° [a straight angle]

5b = 180° – 75° = 105°

$$b = \frac{105^\circ}{5} = 21^\circ$$

∴ a = 4b = 4 × 21° = 84° (vertically opp. ∠s]

Again, 2c + a = 180° [a linear pair]

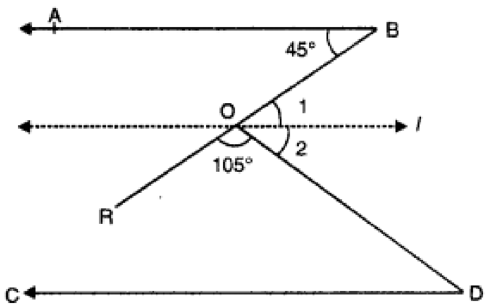
⇒ 2c + 84° = 180°

⇒ 2c = 96°

$$\Rightarrow c = \frac{96^\circ}{2} = 48^\circ$$

Hence, the values of a, b and c are a = 84°, b = 21° and c = 48°.

Ans: 5.



Through O, draw a line 'l' parallel to AB.

⇒ line l will also parallel to CD, then

∠1 = 45°[alternate int. angles]

∠1 + ∠2 + 105° = 180° [straight angle]

∠2 = 180° – 105° – 45°

⇒ ∠2 = 30°

Now, ∠ODC = ∠2 [alternate int. angles]

= ∠ODC = 30°

Ans: 6. In ΔXYZ, we have

∠X + XY + ∠Z = 180°

⇒ ∠Y + ∠Z = 180° – ∠X

⇒ ∠Y + ∠Z = 180° – 72°

⇒ Y + ∠Z = 108°

$$\Rightarrow \frac{1}{2}\angle Y + \frac{1}{2}\angle Z = \frac{1}{2} \times 108^\circ$$

∠OYZ + ∠OZY = 54°

[∵ YO and ZO are the bisector of ∠XYZ and ∠XZY]

$\Rightarrow \angle OYZ + \frac{1}{2} \times 46° = 54°$

∠OYZ + 23° = 54°

⇒ ∠OYZ = 549 – 23° = 31°

In ΔYOZ, we have

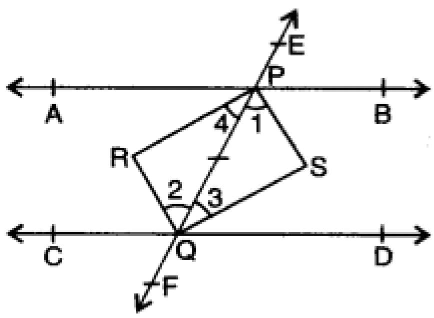
∠YOZ = 180° – (∠OYZ + ∠OZY)

= 180° – (31° + 23°) 180° – 54° = 126°

Long Answer:

Ans: 1. Given: AB | | CD and transversal EF cut them at P and Q respectively and the bisectors of

pair of interior angles form a quadrilateral PRQS



To Prove: PRQS is a rectangle.

Proof: ∵ PS, QR, QS and PR are the bisectors of angles

∠BPQ, ∠CQP, ∠DQP and ∠APQ respectively.

$\therefore \angle 1 = \frac{1}{2} \angle BPQ,\ \angle 2 = \frac{1}{2} \angle CQP,$

$\angle 3 = \frac{1}{2} \angle DQP$ and $\angle 4 = \frac{1}{2} \angle APQ$

Now, AB || CD and EF is a transversal

∴ ∠BPQ = ∠CQP

$\Rightarrow \angle 1 = \angle 2$ ($\because \angle 1\ \frac{1}{2} \angle BPQ$ and $\angle 2 = \frac{1}{2} \angle QP$)

But these are pairs of alternate interior angles of PS and QR

∴ PS | | QR

Similarly, we can prove ∠3 = ∠4 = QS | | PR

∴ PRQS is a parallelogram.

Further $\angle 1 + \angle 3 = \frac{1}{2}\angle BPQ + \frac{1}{2}\angle DQP = \frac{1}{2}(\angle BPQ + \angle DQP)$

$= \frac{1}{2} \times 180° = 90°$ ($\because \angle BPQ + \angle DQP = 180°$)

∴ In ΔPSQ, we have ∠PSQ = 180° − (∠1 + ∠3) = 180° − 90° = 90°

Thus, PRRS is a parallelogram whose one angle ∠PSQ = 90°.

Hence, PRQS is a rectangle.

Ans: 2. Let $\angle B = 2x$ and $\angle C = 2y$

$\because$OB and OC bisect $\angle B$ and $\angle C$ respectively.

$\angle OBC = \frac{1}{2}\angle B = \frac{1}{2} \times 2x = x$

and $\angle OCB = \frac{1}{2}\angle C = \frac{1}{2} \times 2y = y$

Now, in ΔBOC, we have

$\angle BOC + \angle OBC + \angle OCB = 180°$

$\Rightarrow \angle BOC + x + y = 180°$

$\Rightarrow \angle BOC = 180° - (x + y)$

Now, in ΔABC, we have

$\angle A + 2B + C = 180°$

$\Rightarrow \angle A + 2x + 2y = 180°$

$\Rightarrow 2(x + y) = \frac{1}{2}(180° - \angle A)$

$\Rightarrow x + y = 90° - \frac{1}{2}\angle A$ .....(ii)

From (i) and (ii), we have

$\angle BOC = 180° - (90° - \frac{1}{2}\angle A) = 90° + \frac{1}{2}\angle A$

Ans: 3. Here, $\angle 1$ and $\angle 4$ are forming a linear pair

$\angle 1 + \angle 4 = 180°$

$(2x + y)° + (x + 2y)° = 180°$

$3(x + y)° = 180°$

$x + y = 60$

Since l || m and n is a transversal

$\angle 4 = \angle 6$

$(x + 2y)° = (3y + 20)°$

$x - y = 20$

Adding (i) and (ii), we have

$2x = 80 = x = 40$

From (i), we have

$40 + y = 60 \Rightarrow y = 20$

Now, $\angle 1 = (2 \times 40 + 20)° = 100°$

$\angle 4 = (40 + 2 \times 20)° = 80°$

$\angle 8 = \angle 4 = 80°$ [corresponding $\angle$s]

$\angle 1 = \angle 3 = 100°$ [vertically opp. $\angle$s]

$\angle 7 = \angle 3 = 100°$ [corresponding $\angle$s]

Hence, $\angle 7 = 100°$ and $\angle 8 = 80°$

Ans: 4. Here, PQ || SR .

$\Rightarrow \angle PQR = \angle QRT$

$\Rightarrow x + 28° = 65°$

$\Rightarrow x = 65° - 28° = 37°$

Now, in it. ΔSPQ, $\angle P = 90°$

$\therefore \angle P + x + y = 180°$ [angle sum property]

$\therefore 90° + 37° + y = 180°$

$\Rightarrow y = 180° – 90° - 37° = 53°$

Now, $\angle SRQ + \angle QRT = 180°$ [linear pair]

$z + 65° = 180°$

$z = 180° - 65° = 115°$

Ans: 5. In quadrilateral ABCD, we have

$\angle A + \angle B.+ \angle C + \angle D = 360°$

$$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C + \frac{1}{2}\angle D = \frac{1}{2} \times 360°$$

$$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle D = 180° - \frac{1}{2}(\angle B + \angle C)$$

As, AP and DP are the bisectors of $\angle A$ and $\angle D$

$$\therefore \quad \angle PAD = \frac{1}{2}\angle A$$

$$\text{and} \quad \angle PDA = \frac{1}{2}\angle D$$

$$\text{Now,} \quad \angle PAD + \angle PDA = 180° - \frac{1}{2}(\angle B + \angle C) \qquad ...(i)$$

In ΔAPD, we have

$$\angle APD + \angle PAD + \angle PDA = 180°$$

$$\Rightarrow \angle APD + 180° - \frac{1}{2}(\angle B + \angle C) = 180° \qquad \text{[using } (i)\text{]}$$

$$\Rightarrow \angle APD = \frac{1}{2}(\angle B + \angle C)$$

$$\Rightarrow 2\angle APD = \angle B + \angle C$$

Assertion and Reason Answers-

1. d) Assertion is wrong statement but reason is correct statement.

Explanation:

Linear pair

Adjacent angles with opposite rays as noncommon arms are called the linear pair.

They form a straight angle.

Hence **Reason is True.**

Two adjacent angles form a linear pair if non common arms are opposite rays.

If non common sides are not opposite rays then adjacent angles does not form a linear pair.

Hence Assertion "Two adjacent angles always form a linear pair" is False

For example two adjacent angles which are complementary forms a right angle not a linear pair.

2. d) Assertion is wrong statement but reason is correct statement.

Explanation:

ASSERTION : A triangle can have two obtuse angles.

Obtuse angle are the angles whos measure are between 90° and 180°

If a triangle has two obtuse angles then sum of those two angles will be

between (90° + 90°) and (180° + 180°)

= between 180° and 360°

Hence sum of all the angles of triangle would be greater than 180°

But Sum of all the angles of a triangle is 180°

Hence This is not possible

so Assertion is FALSE

REASON : The sum of all the interior angles of a triangle is 180°

TRUE

Case Study Answers-

1.

(i)	(a)	30°
(ii)	(c)	60°
(iii)	(d)	90°
(iv)	(a)	120°
(v)	(b)	180°

2.

(i)	(b)	96°
(ii)	(d)	24°
(iii)	(c)	42°
(iv)	(c)	180°
(v)	(a)	2y + z = 90°