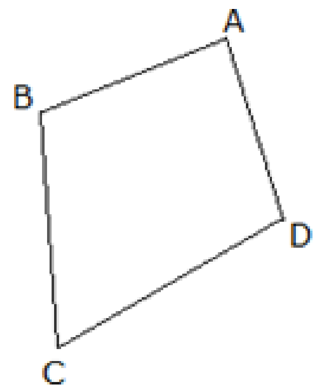
MATHEMATICS

Chapter 8: Quadrilaterals

Quadrilaterals

Quadrilateral

A **quadrilateral** is a closed figure obtained by joining four points (with no three points collinear) in an order.



Here, ABCD is a quadrilateral.

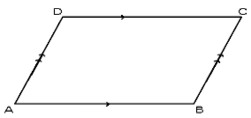
Parts of a quadrilateral

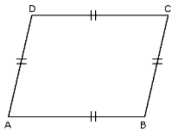
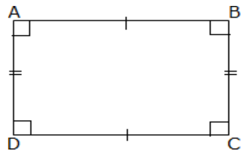
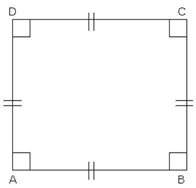
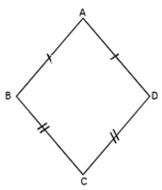
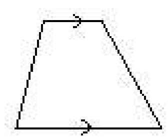
- A quadrilateral has four sides, four angles and four vertices.
- Two sides of a quadrilateral having no common end point are called its **opposite sides**.
- Two sides of a quadrilateral having a common end point are called its **adjacent sides**.
- Two angles of a quadrilateral having common arm are called its **adjacent angles**.
- Two angles of a quadrilateral not having a common arm are called its **opposite angles**.
- A **diagonal** is a line segment obtained on joining the opposite vertices.

Angle sum property of a quadrilateral

Sum of all the angles of a quadrilateral is 360^o. This is known as the **angle sum property of a quadrilateral**.

Types of quadrilaterals and their properties:

Name of a quadrilateral	**Properties**
Parallelogram: A quadrilateral with each pair of opposite sides parallel. 	i. Opposite sides are equal. ii. Opposite angles are equal. iii. Diagonals bisect one another.

Rhombus: A parallelogram with sides of equal length.  	i. All properties of a parallelogram. ii. Diagonals are perpendicular to each other.
Rectangle: A parallelogram with all angles right angle.  	i. All the properties of a parallelogram. ii. Each of the angles is a right angle. iii. Diagonals are equal.
Square: A rectangle with sides of equal length.  	All the properties of a parallelogram, a rhombus and a rectangle.
Kite: A quadrilateral with exactly two pairs of equal consecutive sides.  	i. The diagonals are perpendicular to one another. ii. One of the diagonals bisects the other. iii. If ABCD is a kite, then $\angle B = \angle D$ but $\angle A \neq \angle C$
Trapezium: A quadrilateral with one pair of opposite sides parallel is called trapezium. 	One pair of opposite sides parallel.

Important facts about quadrilaterals

- If the non-parallel sides of trapezium are equal, it is known as **isosceles trapezium**.
- Square, rectangle and rhombus are all parallelograms.
- Kite and trapezium are not parallelograms.
- A square is a rectangle.
- A square is a rhombus.
- A parallelogram is a trapezium.

A quadrilateral is a parallelogram if:

- **i.** each pair of opposite sides of a quadrilateral is equal, or
- **ii.** each pair of opposite angles is equal, or
- **iii.** the diagonals of a quadrilateral bisect other, or
- **iv.** each pair of opposite sides is equal and parallel.

Mid-Point Theorem

The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

Converse of mid-point theorem

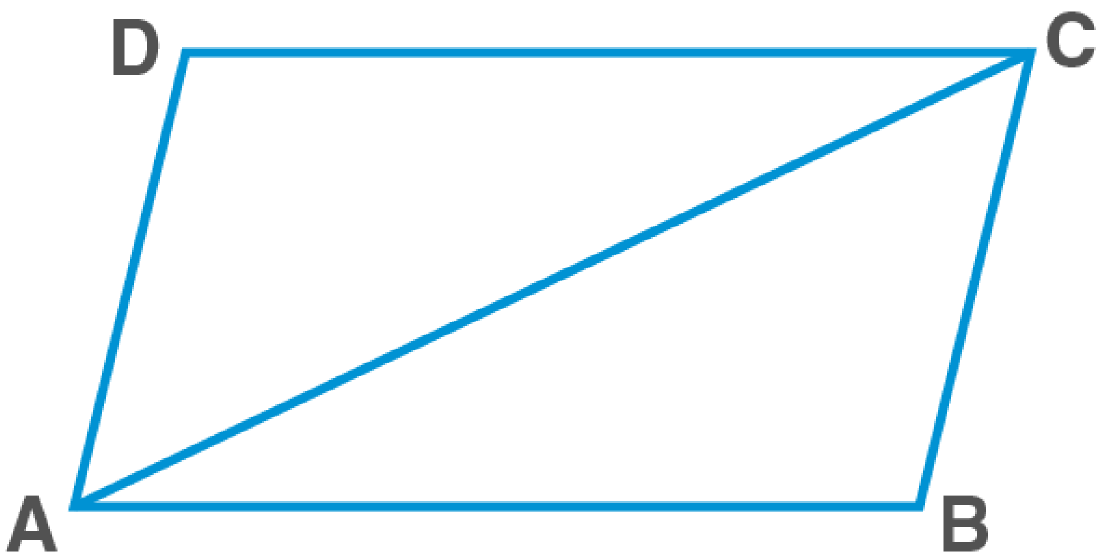
The line drawn through the mid-point of one side of a triangle, parallel to another side, bisects the third side.

Formation of a new quadrilateral using the given data

- If the diagonals of a parallelogram are equal, then it is a rectangle.
- If the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.
- If the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

If there are three or more parallel lines and the intercepts made by them on a transversal are equal, then the corresponding intercepts on any other transversal are also equal.

Parallelogram: Opposite sides of a parallelogram are equal



In ΔABC and ΔCDA

AC = AC [Common / transversal]

∠BCA = ∠DAC [alternate angles]

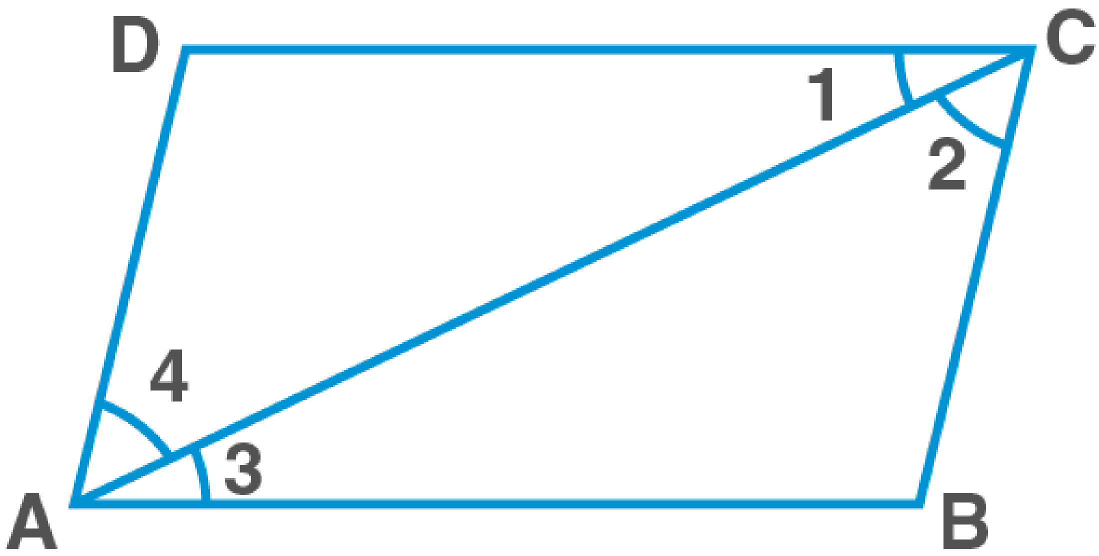
∠BAC = ∠DCA [alternate angles]

ΔABC ≅ ΔCDA [ASA rule]

Hence,

AB = DC and AD= BC [C.P.C.T.C]

Opposite angles in a parallelogram are equal



In parallelogram ABCD

AB || CD; and AC is the transversal

Hence, ∠1 = ∠3 …. (1) (alternate interior angles)

BC || DA; and AC is the transversal

Hence, ∠2 = ∠4 …. (2) (alternate interior angles)

Adding (1) and (2)

∠1 + ∠2 = ∠3 + ∠4

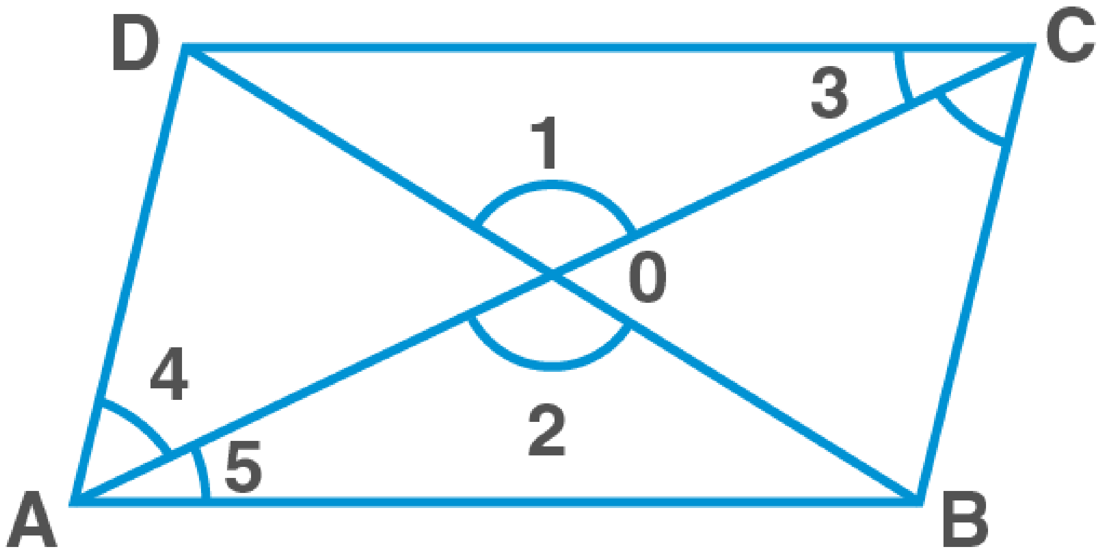
∠BAD = ∠BCD

Similarly,

∠ADC = ∠ABC

Properties of diagonal of a parallelogram

Diagonals of a parallelogram bisect each other.



In ΔAOB and ΔCOD,

∠3 = ∠5 [alternate interior angles]

∠1 = ∠2 [vertically opposite angles]

AB = CD [opp. Sides of parallelogram]

ΔAOB ≅ ΔCOD [AAS rule]

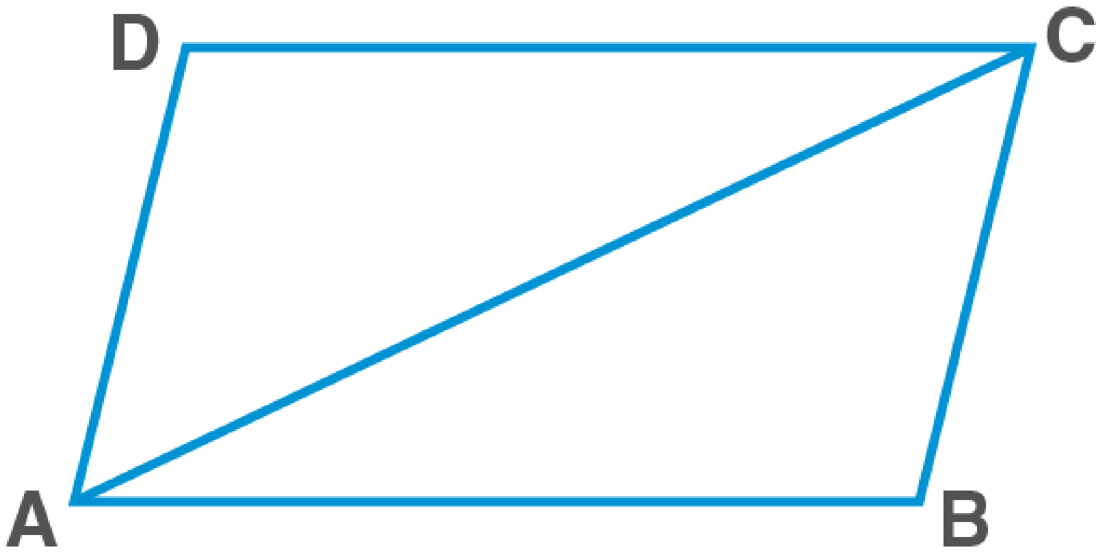
OB = OD and OA = OC [C.P.C.T]

Hence, proved

Conversely,

If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Diagonal of a parallelogram divides it into two congruent triangles.



In ΔABC and ΔCDA,

AB = CD [Opposite sides of parallelogram]

BC = AD [Opposite sides of parallelogram]

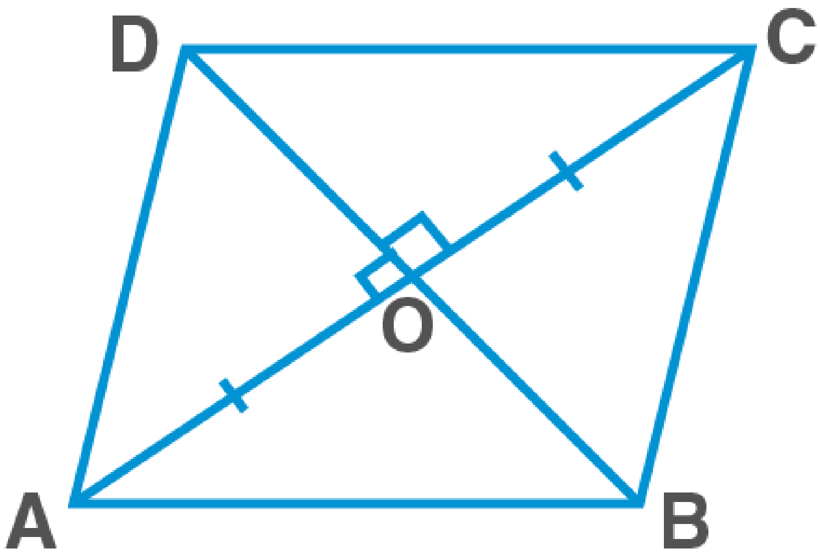
AC = AC [Common side]

ΔABC ≅ ΔCDA [by SSS rule]

Hence, proved.

Diagonals of a rhombus bisect each other at right angles

Diagonals of a rhombus bisect each – other at right angles



In ΔAOD and ΔCOD,

OA = OC [Diagonals of parallelogram bisect each other]

OD = OD [Common side]

AD = CD [Adjacent sides of a rhombus]

ΔAOD ≅ ΔCOD [SSS rule]

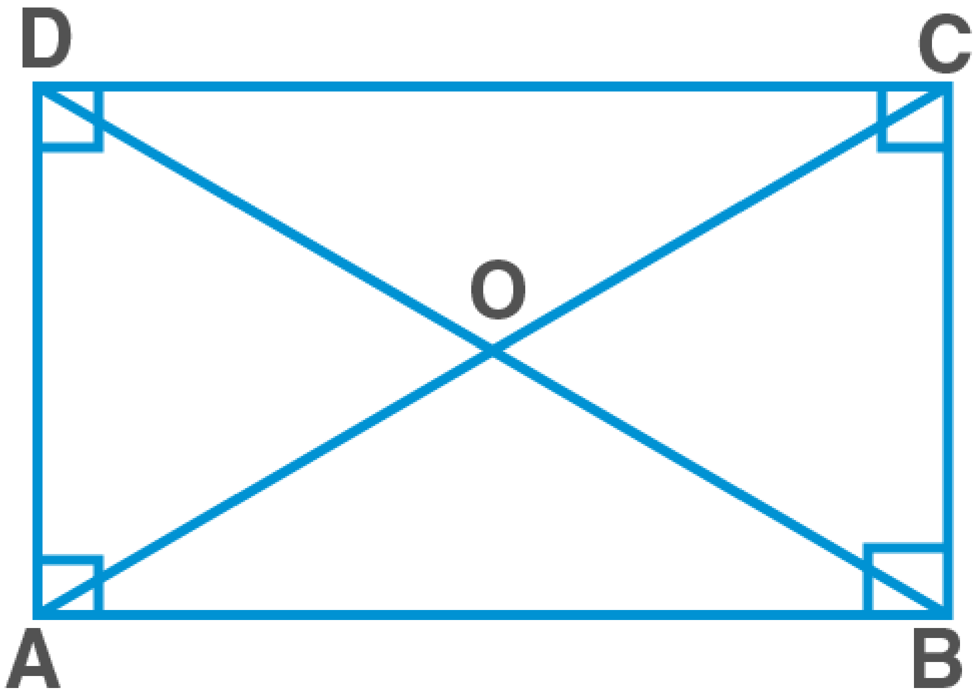
∠AOD = ∠DOC [C.P.C.T]

∠AOD + ∠DOC = 180 [∵ AOC is a straight line]

Hence, ∠AOD = ∠DOC = 90

Hence proved.

Diagonals of a rectangle bisect each other and are equal



Rectangle ABCD

In ΔABC and ΔBAD,

AB = BA [Common side]

BC = AD [Opposite sides of a rectangle]

∠ABC = ∠BAD [Each = 900 ∵ ABCD is a Rectangle]

ΔABC ≅ ΔBAD [SAS rule]

∴AC = BD [C.P.C.T]

Consider ΔOAD and ΔOCB,

AD = CB [Opposite sides of a rectangle]

∠OAD = ∠OCB [∵ AD||BC and transversal AC intersects them]

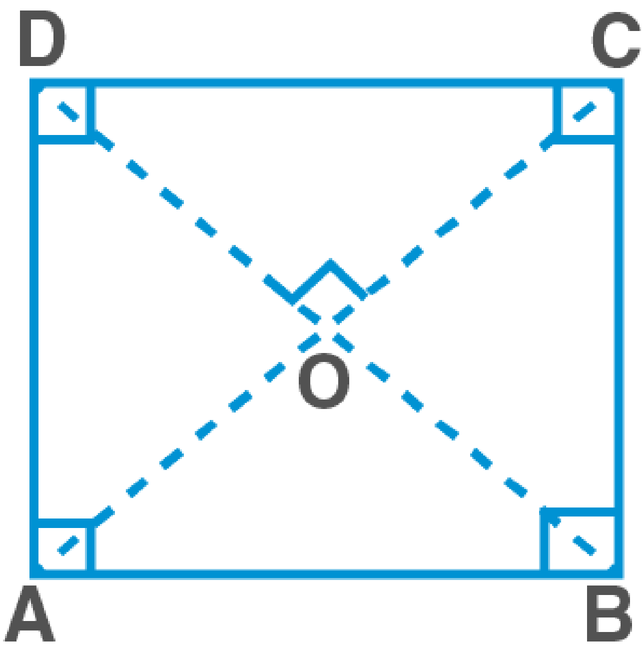
∠ODA = ∠OBC [∵ AD||BC and transversal BD intersects them]

ΔOAD ≅ ΔOCB [ASA rule]

∴OA = OC [C.P.C.T]

Similarly, we can prove OB=OD

Diagonals of a square bisect each other at right angles and are equal



In ΔABC and ΔBAD,

AB = BA [Common side]

BC = AD [Opposite sides of a Square]

∠ABC = ∠BAD [Each = 900 ∵ ABCD is a Square]

ΔABC ≅ ΔBAD [SAS rule]

∴ AC = BD [C.P.C.T]

Consider ΔOAD and ΔOCB,

AD = CB [Opposite sides of a Square]

∠OAD = ∠OCB [∵ AD||BC and transversal AC intersects them]

∠ODA = ∠OBC [∵ AD||BC and transversal BD intersects them]

ΔOAD ≅ ΔOCB [ASA rule]

∴ OA = OC [C.P.C.T]

Similarly, we can prove OB=OD

In ΔOBA and ΔODA,

OB = OD [proved above]

BA = DA [Sides of a Square]

OA = OA [Common side]

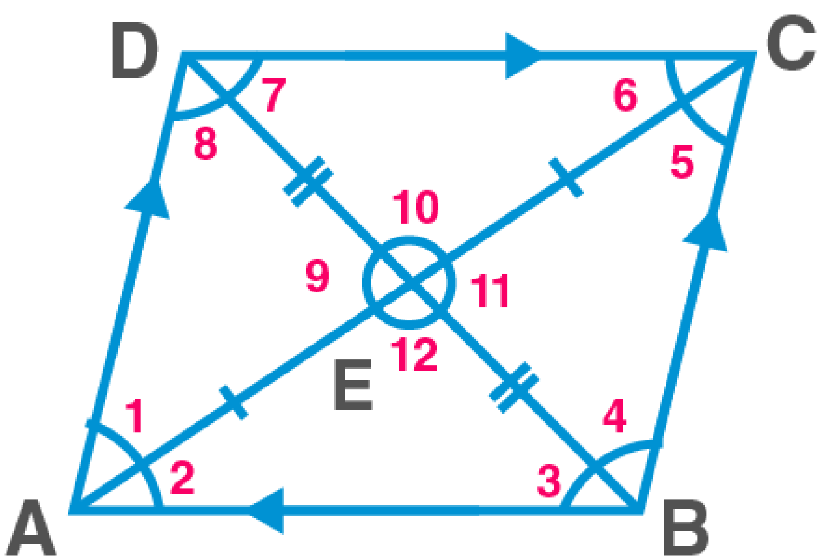
ΔOBA ≅ ΔODA, [SSS rule]

∴ ∠AOB = ∠AOD [C.P.C.T]

But ∠AOB + ∠AOD = 1800 [Linear pair]

∴ ∠AOB = ∠AOD = 90^0

Important results related to parallelograms



Opposite sides of a parallelogram are parallel and equal.

AB||CD, AD||BC, AB = CD, AD = BC

Opposite angles of a parallelogram are equal adjacent angels are supplementary.

$\angle A = \angle C$, $\angle B = \angle D$,

$\angle A + \angle B = 1800$, $\angle B + \angle C = 1800$, $\angle C + \angle D = 1800$, $\angle D + \angle A = 1800$

A diagonal of parallelogram divides it into two congruent triangles.

$\Delta ABC \cong \Delta CDA$ [With respect to AC as diagonal]

$\Delta ADB \cong \Delta CBD$ [With respect to BD as diagonal]

The diagonals of a parallelogram bisect each other.

AE = CE, BE = DE

$\angle 1 = \angle 5$ (alternate interior angles)

$\angle 2 = \angle 6$ (alternate interior angles)

$\angle 3 = \angle 7$ (alternate interior angles)

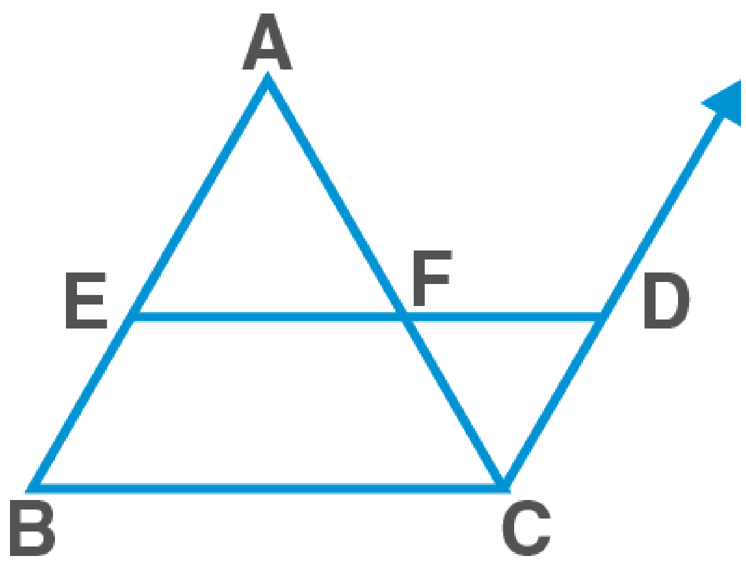
$\angle 4 = \angle 8$ (alternate interior angles)

$\angle 9 = \angle 11$ (vertically opp. angles)

$\angle 10 = \angle 12$ (vertically opp. angles)

The Mid-Point Theorem

The line segment joining the midpoints of two sides of a triangle is parallel to the third side and is half of the third side



In ΔABC, E – the midpoint of AB; F – the midpoint of AC

Construction: Produce EF to D such that EF = DF.

In ΔAEF and ΔCDF,

AF = CF [F is the midpoint of AC]

∠AFE = ∠CFD [V.O.A]

EF = DF [Construction]

∴ ΔAEF ≅ ΔCDF [SAS rule]

Hence,

∠EAF = ∠DCF …. (1)

DC = EA = EB [E is the midpoint of AB]

DC || EA || AB [Since, (1), alternate interior angles]

DC || EB

So EBCD is a parallelogram

Therefore, BC = ED and BC || ED

Since ED = EF + FD = 2EF = BC [∵ EF=FD]

We have, EF = 12 BC and EF||BC

Hence proved.

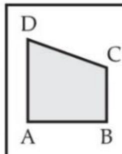
CHAPTER : 8 QUADRILATERALS

Quadrilaterals

Quadrilateral

Figure formed by joining four points in an order

It has four - vertices, angles and sides each

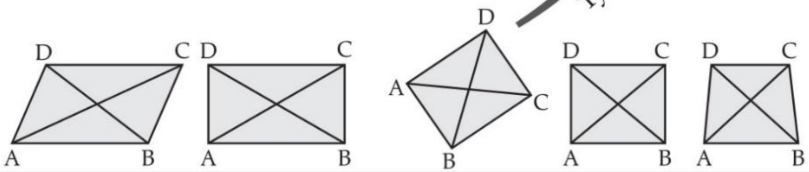


ABCD is a Quadrilateral

Mid-point theorem

Statement	Figure
The line-segment joining the mid-points of two sides of a triangle is parallel to the third side.	If E and F are mid-point of AB and AC, then EF ∣∣ BC
The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side	If E is the mid-point of AB EF ∣∣ BC, then AF = FC, F is the mid-point of AC

Types



Property	Parallelogram	Rectangle	Rhombus	Square	Trapezium
All sides are congruent	No	No	Yes	Yes	No
Opposite sides are parallel and congruent	Yes	Yes	Yes	Yes	Parallel but not congruent
All angles are congruent	No	Yes	No	Yes	No
Opposite angles are congruent	Yes	Yes	Yes	Yes	Yes
Diagonals are congruent	No	Yes	No	Yes	Yes
Diagonals are perpendicular	No	No	Yes	Yes	No
Diagonals bisect each other	Yes	Yes	Yes	Yes	No
Adjacent angles are supplementary	Yes	Yes	Yes	Yes	Yes

Properties

Statement	Figure
1. A diagonal of a parallelogram divides it into two congruent triangles.	ABCD is a parallelogram AC diagonal then $\Delta ABC \cong \Delta ADC$
2. In a parallelogram, opposite sides are equal and parallel.	In parallelogram ABCD, AB ∣∣ CD, AD ∣∣ BC, and AB = CD, AD = BC
3. If each pair of opposite sides of a quadrilateral are equal and parallel, then it is a parallelogram.	If AB ∣∣ CD, AD ∣∣ BC, and AB = CD, AD = BC then ABCD is a parallelogram
4. In a parallelogram, opposite angles are equal.	In parallelogram ABCD, $\angle A = \angle C$, $\angle B = \angle D$
5. If in a quadrilateral, each pair of opposite angle is equal, then it is a parallelogram.	If $\angle A = \angle C$, $\angle B = \angle D$ then ABCD is a parallelogram
6. The diagonals of a parallelogram bisect each other.	In parallelogram ABCD, OA = OC then OB = OD
7. If the diagonals of a quadrilateral bisect each other, then it is parallelogram.	If OA = OC OB = OD then ABCD is parallelogram

Important Questions

Multiple Choice questions-

Question 1. A diagonal of a Rectangle is inclines to one side of the rectangle at an angle of 25°. The Acute Angle between the diagonals is:

(a) 115°

(b) 50°

(c) 40°

(d) 25°

Question 2. The diagonals of a rectangle PQRS intersects at O. If ∠QOR = 44°, ∠OPS =?

(a) 82°

(b) 52°

(c) 68°

(d) 75°

Question 3. If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3:7:6:4, then ABCD is

(a) Rhombus

(b) Parallelogram

(c) Trapezium

(d) Kite

Question 4. All the angles of a convex quadrilateral are congruent. However, not all its sides are congruent. What type of quadrilateral is it?

(a) Parallelogram

(b) Square

(c) Rectangle

(d) Trapezium

Question 5. In a Quadrilateral ABCD, AB = BC and CD = DA, then the quadrilateral is a

(a) Triangle

(b) Kite

(c) Rhombus

(d) Rectangle

Question 6. The angles of a quadrilateral are $(5x)°$, $(3x + 10)°$, $(6x – 20)°$ and $(x + 25)°$. Now, the measure of each angle of the quadrilateral will be

(a) 115°, 79°, 118°, 48°

(b) 100° 79°, 118°, 63°

(c) 110°, 84°, 106°, 60°

(d) 75°, 89°, 128°, 68°

Question 7. The diagonals of rhombus are 12 cm and 16 cm. The length of the side of rhombus is:

(a) 12cm

(b) 16cm

(c) 8cm

(d) 10cm

Question 8. In quadrilateral PQRS, if $\angle P = 60°$ and $\angle Q : \angle R : \angle S = 2 : 3 : 7$, then $\angle S =$

(a) 175°

(b) 210°

(c) 150°

(d) 135°

Question 9. In parallelogram ABCD, if $\angle A = 2x + 15°$, $\angle B = 3x – 25°$, then value of x is:

(a) 91°

(b) 89°

(c) 34°

(d) 38°

Question 10. If ABCD is a trapezium in which AB || CD and AD = BC, then:

(a) $\angle A = \angle B$

(b) $\angle A > \angle B$

(c) $\angle A < \angle B$

(d) None of the above

Very Short:

1. If one angle of a parallelogram is twice of its adjacent angle, find the angles of the parallelogram.

2. If the diagonals of a quadrilateral bisect each other at right angles, then name the

quadrilateral.

3. Three angles of a quadrilateral are equal, and the fourth angle is equal to 144o. Find each of the equal angles of the quadrilateral.

4. If ABCD is a parallelogram, then what is the measure of ∠A – ∠C?

5. PQRS is a parallelogram, in which PQ = 12 cm and its perimeter is 40 cm. Find the length of each side of the parallelogram.

6. Two consecutive angles of a parallelogram are (x + 60)° and (2x + 30)°. What special name can you give to this parallelogram?

7. ONKA is a square with ∠KON = 45°. Determine ∠KOA.

8. In quadrilateral PQRS, if ∠P = 60° and ∠Q : ∠R : ∠S = 2 : 3 : 7, then find the measure of ∠S.

Short Questions:

1. ABCD is a parallelogram in which ∠ADC = 75° and side AB is produced to point E as shown in the figure. Find x + y.
2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.
3. In the figure, ABCD is a rhombus, whose diagonals meet at O. Find the values of x and y.
4. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see fig.). Show that:

   (i) ΔAPB = ΔCQD

   (ii) AP = CQ
5. The diagonals of a quadrilateral ABCD are perpendicular to each other. Show that the quadrilateral formed by joining the mid-points of its sides is a rectangle.

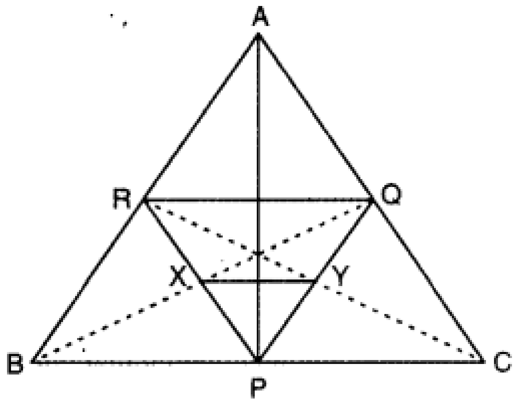
   

6. In the fig., D, E and F are, respectively the mid-points of sides BC, CA and AB of an equilateral triangle ABC. Prove that DEF is also an equilateral triangle.

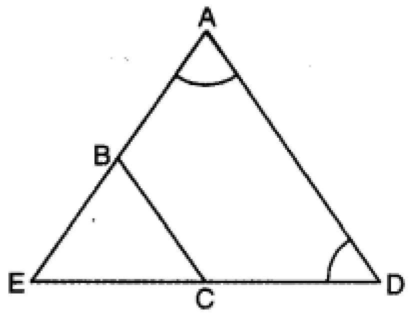
   B D C

Long Questions:

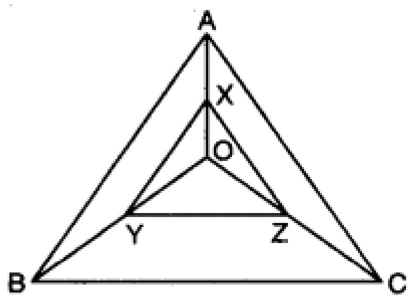
1. In the figure, P, Q and R are the mid-points of the sides BC, AC and AB of ΔABC. If BQ and PR intersect at X and CR and PQ intersect at Y, then show that $XY = \frac{1}{4}\,BC$



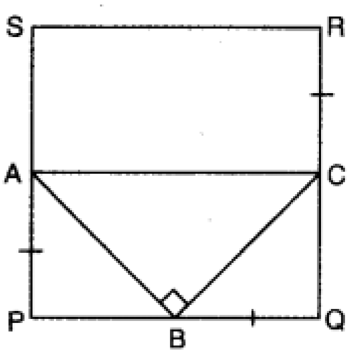
2. In the given figure, AE = DE and BC || AD. Prove that the points A, B, C and D are concyclic. Also, prove that the diagonals of the quadrilateral ABCD are equal.



3. In ΔABC, AB = 8cm, BC = 9 cm and AC = 10cm. X, Y and Z are mid-points of AO, BO and CO respectively as shown in the figure. Find the lengths of the sides of ΔXYZ.



4. PQRS is a square and ∠ABC = 90° as shown in the figure. If AP = BQ = CR, then prove that ∠BAC = 45°



5. ABCD is a parallelogram. If the bisectors DP and CP of angles D and C meet at P on side AB, then show that P is the mid-point of side AB.

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: ABCD is a square. AC and BD intersect at O. The measure of +AOB = 90°.

Reason: Diagonals of a square bisect each other at right angles.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: The consecutive sides of a quadrilateral have one common point.

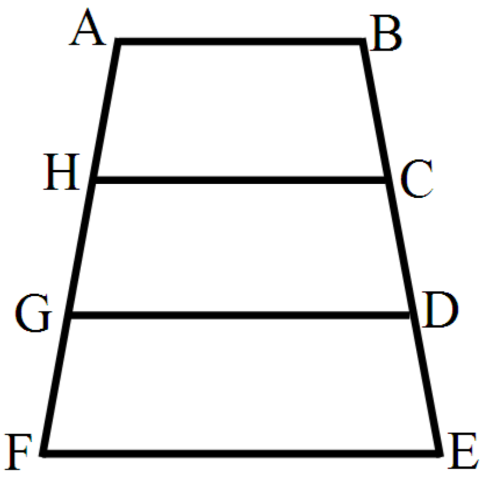
Reason: The opposite sides of a quadrilateral have two common point.

Case Study Questions-

1. Read the Source/ Text given below and answer these questions:

Sohan wants to show gratitude towards his teacher by giving her a card made by him. He has three pieces of trapezium pasted one above the other as shown in fig. These pieces are arranged in a way that AB || HC || GD || FE. Also BC =

CD = DE and AH = HG = GF = 6cm. He wants to decorate the card by putting up a colored tape on the nonparallel sides of the trapezium.



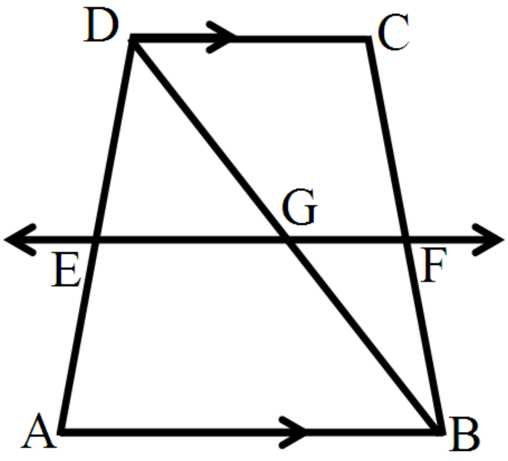
i. Find the total length of colored tape required if DE = 4cm.
   a. 20cm
   b. 30cm
   c. 40cm
   d. 50cm

ii. ABHC is a trapezium in which AB || HC and ∠A=∠B=45∘. Find angles C and H of the trapezium.
   a. 135, 130
   b. 130, 135
   c. 135, 135
   d. 130, 130

iii. What is the difference between trapezium and parallelogram?
   a. Trapezium has 2 sides, and parallelogram has 4 sides.
   b. Trapezium has 4 sides, and parallelogram has 2 sides.
   c. Trapezium has 1 pair of parallel sides, and parallelogram has 2 pairs of parallel sides.
   d. Trapezium has 2 pairs of parallel sides, and parallelogram has 1 pair of parallel sides.

iv. Diagonals in isosceles trapezoid are _________.
   a. parallel.
   b. opposite.
   c. vertical.
   d. equal.

v. ABCD is a trapezium where AB || DC, BD is the diagonal and E is the midpoint of AD. A line is drawn through E parallel to AB intersecting BC at F. Which of these is true?



a. BF = FC

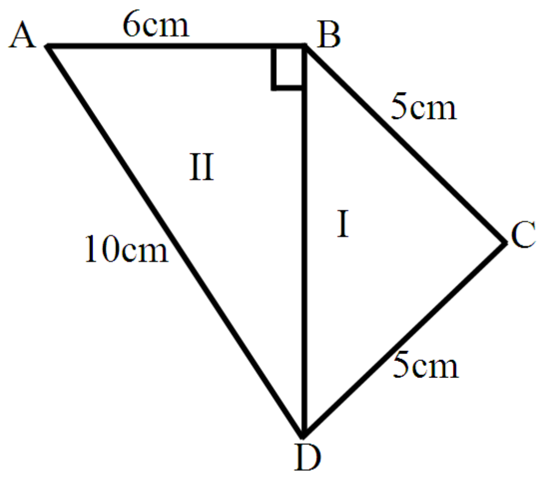
b. EA = FB

c. CF = DE

d. None of these

2. Read the Source/ Text given below and answer any four questions:

Chocolate is in the form of a quadrilateral with sides 6cm and 10cm, 5cm and 5cm(as shown in the figure) is cut into two parts on one of its diagonal by a lady. Part-I is given to her maid and part II is equally divided among a driver and gardener.



i. Length of BD:
   a. 9cm
   b. 8cm
   c. 7cm
   d. 6cm
ii. Area of △ABC:
   a. 24cm^2
   b. 12cm^2
   c. 42cm^2
   d. 21cm^2
iii. The sum of all the angles of a quadrilateral is equal to:
   a. 180°
   b. 270°
   c. 360°
   d. 90°
iv. A diagonal of a parallelogram divides it into two congruent:
   a. Square.
   b. Parallelogram.
   c. Triangles.
   d. Rectangle.
v. Each angle of the rectangle is:
   a. More than 90°
   b. Less than 90°
   c. Equal to 90°

d. Equal to 45°

Answer Key:

MCQ:

1. (b) 50°
2. (c) 68°
3. (c) Trapezium
4. (c) Rectangle
5. (b) Kite
6. (a) 115°, 79°, 118°, 48°
7. (d) 10cm
8. (a) 175°
9. (d) 38°
10. (a) $\angle A = \angle B$

Very Short Answer:

1. Let the two adjacent angles be x and 2x.

   In a parallelogram, sum of the adjacent angles are 180°

   $\therefore x + 2x = 180°$

   $\Rightarrow 3x = 180°$

   $\Rightarrow x = 60°$

   Thus, the two adjacent angles are 120° and 60°. Hence, the angles of the parallelogram are 120°, 60°, 120° and 60°.

2. Rhombus.

3. Let each equal angle of given quadrilateral be x.

   We know that sum of all interior angles of a quadrilateral is 360°

   $\therefore x + x + x + 144° = 360°$

   $3x = 360° – 144°$

   $3x = 216°$

   $x = 72°$

   Hence, each equal angle of the quadrilateral is of 72o measures.

4. $\angle A – \angle C = 0°$ (opposite angles of parallelogram are equal]

5.

Here, PQ = SR = 12 cm

Let PS = x and PS = QR

$\therefore x + 12 + x + 12 = \text{Perimeter}$

2x + 24 = 40

2x = 16

x= 8

Hence, length of each side of the parallelogram is 12cm, 8 cm, 12cm and 8cm.

6. We know that consecutive interior angles of a parallelogram are supplementary.

∴ (x + 60° + (2x + 30)° = 180°

⇒ 3x° + 90° = 180°

⇒ 3x° = 90°

⇒ x° = 30°

Thus, two consecutive angles are (30 + 60)°, 12 x 30 + 30)”. i.e., 90° and 90°.

Hence, the special name of the given parallelogram is rectangle.

7. Since ONKA is a square

∴ ∠AON = 90°

We know that diagonal of a square bisects its ∠s

⇒ ∠AOK = ∠KON = 45°

Hence, ∠KOA = 45°

Now, ∠A + ∠B + ∠C = 180°

⇒ ∠A + 70° + 70° = 180°

[∵ ∠B = 70°]

⇒ ∠A = 180° – 70° – 70° = 40°

8. Let ∠Q = 2x, ∠R = 3x and ∠S = 7x

Now, ∠P + ∠Q + ∠R + ∠S = 360°

⇒ 60° + 2x + 3x + 7x = 360°

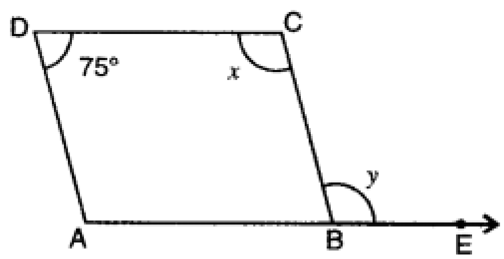
⇒ 12x = 300°

$X = \frac{300°}{12} = 25°$

∠S = 7x = 7 x 25° = 175°

Short Answer:

Ans: 1.



Here, ∠C and ∠D are adjacent angles of the parallelogram.

$\therefore \angle C + \angle D = 180°$

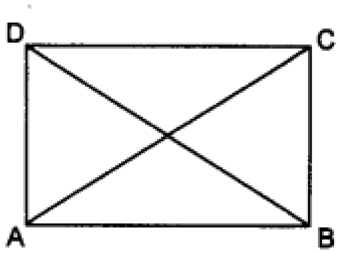
$\Rightarrow x + 75° = 180°$

$\Rightarrow x = 105°$

Also, $y = x = 105°$ [alt. int. angles]

Thus, $x + y = 105° + 105° = 210°$

Ans: 2.



Given: A parallelogram ABCD, in which AC = BD.

To Prove: ΔBCD is a rectangle.

Proof: In ΔABC and ΔBAD

AB = AB (common]

AC = BD (given]

BC = AD (opp. sides of a ||gm]

$\Rightarrow \Delta ABC \cong \Delta BAD$

[by SSS congruence axiom]

$\Rightarrow \angle ABC = \angle BAD$ (c.p.c.t.)

Also, $\angle ABC + \angle BAD = 180°$ (co-interior angles)

$\angle ABC + \angle ABC = 180°$ [$\because \angle ABC = \angle BAD$]

$2\angle ABC = 180°$

$\angle ABC = 1/2 \times 180° = 90°$

Hence, parallelogram ABCD is a rectangle.

Ans: 3. Since diagonals of a rhombus bisect each other at right angle.

In $\therefore$ ΔAOB, we have

$\angle OAB + \angle x + 90° = 180°$

$\angle x = 180° – 90° – 35°$

$= 55°$

Also,

$\angle DAO = \angle BAO = 35°$

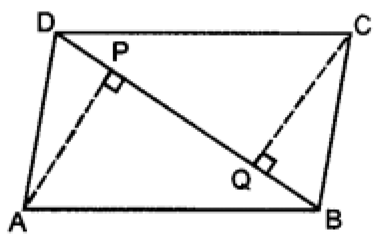
$\angle y + \angle DAO + \angle BAO + \angle x = 180°$

$\Rightarrow \angle y + 35° + 35° + 55° = 180°$

$\Rightarrow \angle y = 180° – 125o = 55°$

Hence, the values of x and y are x = 55°, y = 55

Ans: 4.



Given: In ||gm ABCD, AP and CQ are perpendiculars from the

vertices A and C on the diagonal BD.

To Prove: (i) $\Delta APB \cong \Delta CQD$

(ii) AP = CQ

Proof: (i) In ΔAPB and ΔCQD

AB = DC (opp. sides of a ||gm ABCD]

$\angle APB = \angle DQC$ (each = 90°)

$\angle ABP = \angle CDQ$ (alt. int. $\angle$s]

$\Rightarrow \Delta APB \cong \Delta CQD$ [by AAS congruence axiom]

(ii) $\Rightarrow$ AP = CQ [c.p.c.t.]

Ans: 5. Given: A quadrilateral ABCD whose diagonals AC and BD are perpendicular to each other at O. P, Q, R and S are mid-points of side AB, BC, CD and DA respectively are joined are formed quadrilateral PQRS.

To Prove: PQRS is a rectangle.

Proof: In ΔABC, P and Q are mid-points of AB and BC respectively.

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$ AC ... (i) (mid-point theorem]

Further, in SACD, R and S are mid-points of CD and DA respectively.

SR || AC and SR = $\frac{1}{2}$ AC ... (ii) (mid-point theorem]

From (i) and (ii), we have PQ || SR and PQ = SR

Thus, one pair of opposite sides of quadrilateral PQRS are parallel and equal.

$\therefore$ PQRS is a parallelogram.

Since PQ|| AC PM || NO

In ΔABD, P and S are mid-points of AB and AD respectively.

PS || BD (mid-point theorem]

$\Rightarrow$ PN || MO

$\therefore$ Opposite sides of quadrilateral PMON are parallel.

$\therefore$ PMON is a parallelogram.

$\angle$ MPN = $\angle$ MON (opposite angles of ||gm are equal]

But $\angle$MON = 90° [given]

$\therefore$ $\angle$MPN = 90° $\Rightarrow$ $\angle$QPS = 90°

Thus, PQRS is a parallelogram whose one angle is 90°

$\therefore$ PQRS is a rectangle.

Ans: 6. Since line segment joining the mid-points of two sides of a triangle is half of the third side.

Therefore, D and E are mid-points of BC and AC respectively.

$\Rightarrow$ DE = $\frac{1}{2}AB$ ...(i)

E and F are the mid-points of AC and AB respectively.

$\therefore$ EF = $\frac{1}{2}BC$ ... (ii)

F and D are the mid-points of AB and BC respectively.

$\therefore$ FD = $\frac{1}{2}$ AC ... (iii)

Now, SABC is an equilateral triangle.

$\Rightarrow$ AB = BC = CA

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}BC = \frac{1}{2}CA$

$\Rightarrow$ DE = EF = FD (using (i), (ii) and (iii)]

Hence, DEF is an equilateral triangle

Long Answer:

Ans: 1. Here, in ΔABC, R and Q are the mid-points of AB and AC respectively.

$\therefore$ By using mid-point theorem, we have

RQ || BC and RQ = $\frac{1}{2}$ BC

$\therefore$ RQ = BP = PC [$\because$ P is the mid-point of BC]

$\therefore$ RQ || BP and RQ || PC

In quadrilateral BPQR

RQ || BP, RQ = BP (proved above)

$\therefore$ BPQR is a parallelogram. [$\because$ one pair of opp. sides is parallel as well as equal]

$\therefore$ X is the mid-point of PR. [$\because$ diagonals of a ||gm bisect each other]

Now, in quadrilateral PCQR

RQ || PC and RQ = PC [proved above)

∴ PCQR is a parallelogram [∵ one pair of opp. sides is parallel as well as equal]

∴ Y is the mid-point of PQ [∵ diagonals of a ||gm bisect each other]

In ΔPQR

∴ X and Y are mid-points of PR and PQ respectively.

$\therefore XY \parallel RQ$ and $XY = \frac{1}{2}RQ$ [by using mid-point theorem]

$XY = \frac{1}{2}\left(\frac{1}{2}BC\right)$ $\left[\because RQ = \frac{1}{2}BC\right]$

$\Rightarrow XY = \frac{1}{4}BC$

Ans: 2. Since AE = DE

∠D = ∠A .... (i) [∵ ∠s opp. to equal sides of a Δ]

Again, BC || AD

∠EBC = ∠A .... (ii) (corresponding ∠s]

From (i) and (ii), we have

∠D = ∠EBC .... (iii)

But ∠EBC + ∠ABC = 180° (a linear pair]

∠D + ∠ABC = 180° (using (iii)]

Now, a pair of opposite angles of quadrilateral ABCD is supplementary

Thus, ABCD is a cyclic quadrilateral i.e., A, B, C and D'are concyclic. In ΔABD and ΔDCA

∠ABD = ∠ACD [∠s in the same segment for cyclic quad. ABCD]

∠BAD = ∠CDA [using (i)]

AD = AD (common]

So, by using AAS congruence axiom, we have

ΔABD ≅ ΔDCA

Hence, BD = CA [c.p.c.t.]

Ans: 3. Here, in ΔABC, AB = 8cm, BC = 9cm, AC = 10cm.

In ΔAOB, X and Y are the mid-points of AO and BO.

∴ By using mid-point theorem, we have

$XY = \frac{1}{2}AB = \frac{1}{2} \times 8cm = 4cm$

Similarly, in ΔτBOC, Y and Z are the mid-points of BO and CO.

∴ By using mid-point theorem, we have

$YZ = \frac{1}{2}BC = \frac{1}{2} \times 9cm = 4.5cm$

And, in ΔτCOA, Z and X are the mid-points of CO and AO.

$\therefore ZX = \frac{1}{2} AC = \frac{1}{2} \times 10cm = 5cm$

Hence, the lengths of the sides of ΔXYZ are $XY = 4cm$, $YZ = 4.5$ cm and $ZX = 5cm$.

Ans: 4. Since PQRS is a square.

$\therefore PQ = QR$ ... (I) [$\because$ sides of a square are equal]

Also, $BQ = CR$ ... (ii) [given]

Subtracting (ii) from (i), we obtain

$PQ - BQ = QR - CR$

$\Rightarrow PB = QC$ ... (iii)

In $\Delta\tau APB$ and $\Delta\tau BQC$

$AP = BQ$

[given $\angle APB = \angle BQC = 90°$](each angle of a square is 90°)

$PB = QC$ (using (iii)]

So, by using SAS congruence axiom, we have

$\Delta APB \cong \Delta BQC$

$\therefore AB = BC$ [c.p.c.t.]

Now, in ΔABC

$AB = BC$ [proved above]

$\therefore \angle ACB = \angle BAC = x°$ (say) [$\angle$s opp. to equal sides]

Also, $\angle B + \angle ACB + \angle BAC = 180°$

$\Rightarrow 90° + x + x = 180°$

$\Rightarrow 2x° = 90°$

$x° = 45°$

Hence, $\angle BAC = 45°$

Ans: 5.

Since DP and CP are angle bisectors of $\angle D$ and $\angle C$ respectively.

: $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

Now, AB || DC and CP is a transversal

$\therefore \angle 5 = \angle 1$ [alt. int. $\angle$s]

But $\angle 1 = \angle 2$ [given]

$\therefore \angle 5 = \angle 2$

Now, in ABCP, $\angle 5 = \angle 2$

$\Rightarrow BC = BP$ ... (I) [sides opp. to equal $\angle$s of a A]

Again, AB || DC and DP is a transversal.

$\therefore \angle 6 = \angle 3$ (alt. int. Δs]

But $\angle 4 = \angle 3$ [given]

$\therefore \angle 6 = \angle 4$

Now, in ΔADP, $\angle 6 = \angle 4$

⇒ DA = AP …. (ii) (sides opp. to equal ∠s of a A]

Also, BC = DA… (iii) (opp. sides of parallelogram)

From (i), (ii) and (iii), we have

BP = AP

Hence, P is the mid-point of side AB.

Assertion and Reason Answers-

1. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

2. c) Assertion is correct statement but reason is wrong statement.

Case Study Answers-

1.

(i)	(b)	30cm
(ii)	(c)	135, 135
(iii)	(c)	Trapezium has 1 pair of parallel sides, and parallelogram has 2 pairs of parallel sides.
(iv)	(d)	equal.
(v)	(a)	BF = FC

2.

(i)	(b)	8cm
(ii)	(a)	$24cm^2$
(iii)	(c)	360°
(iv)	(c)	Triangles.
(v)	(c)	Equal to 90°