MATHEMATICS

Chapter 7: Triangles

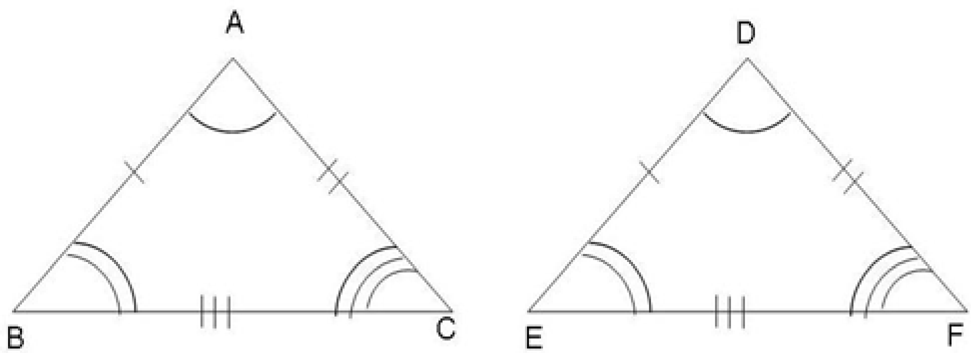
Triangles

What are congruent figures

- Two figures are said to be **congruent** if they are of the same shape and of the same size.
- Two circles of the same radii are congruent.
- Two squares of the same sides are congruent.

Congruent triangles

If two triangles ABC and DEF are congruent under the correspondence A ↔ D, B ↔ E and C ↔ F, then symbolically, it is expressed as $\Delta ABC \cong \Delta DEF$.

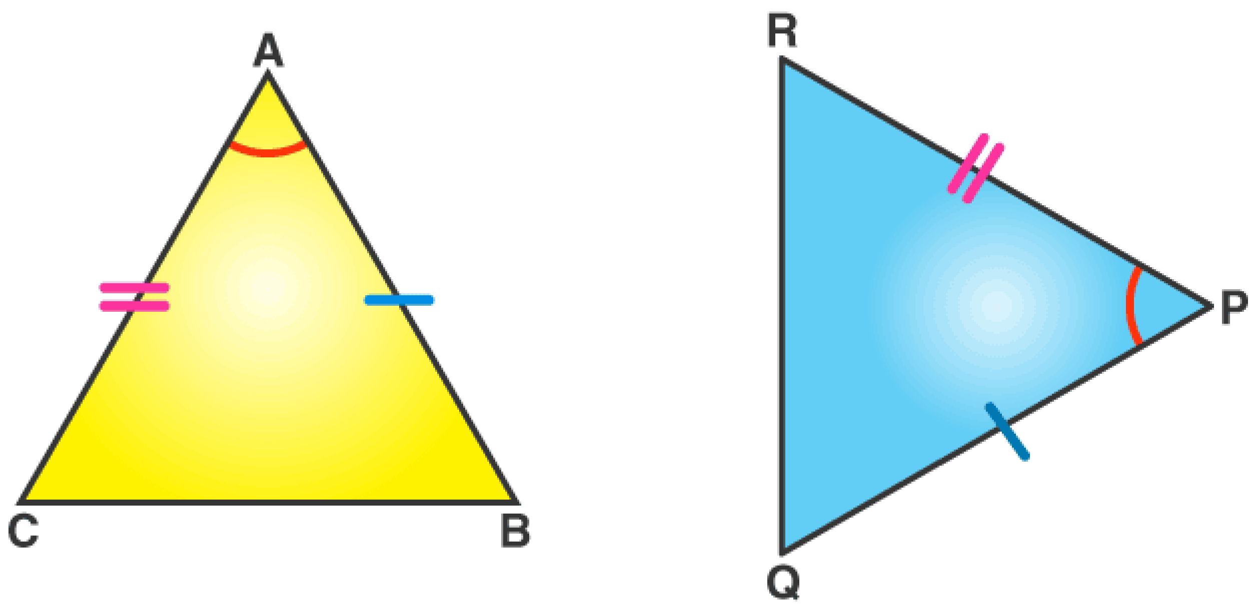


In congruent triangles, **corresponding parts are equal**. We write in short 'CPCT' for corresponding parts of congruent triangles.

SAS (Side – Angle – Side) congruence rule

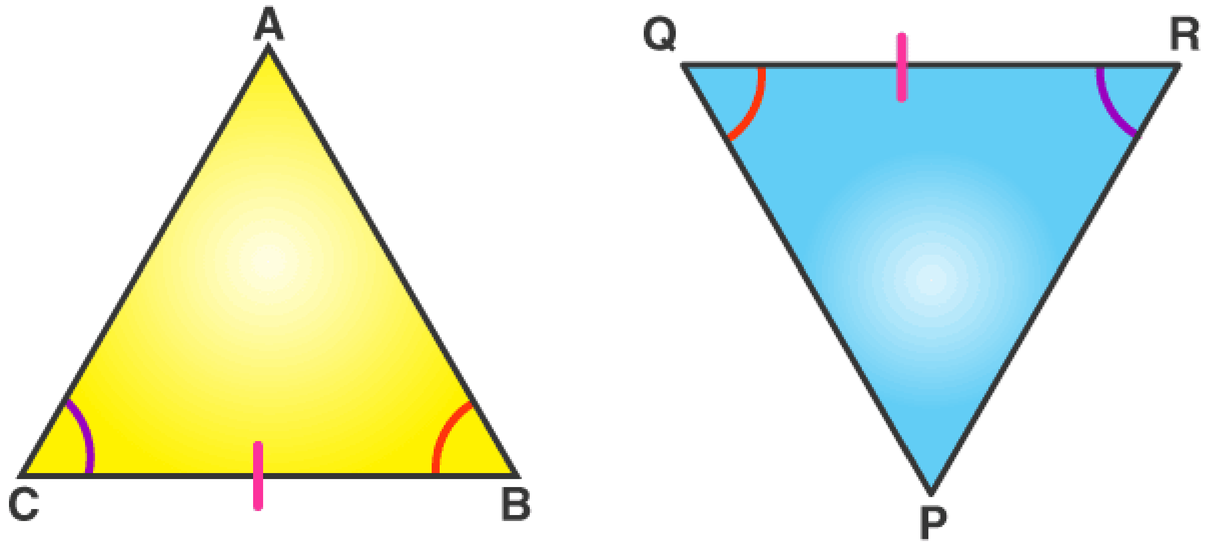
Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.

Note: SAS congruence rule holds but not ASS or SSA rule.



ASA (Angle – Side – Angle) congruence rule

Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle.



AAS (Angle – Angle – Side) congruence rule

Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.

AAS congruency can be proved in easy steps. Suppose we have two triangles ABC and DEF, where,

∠B = ∠E [Corresponding sides] ∠C = ∠F [Corresponding sides] And

AC = DF [Adjacent sides]

By angle sum property of triangle, we know that;

∠A + ∠B + ∠C = 180 ....(1)

∠D + ∠E + ∠F = 180 ....(2)

From equation 1 and 2 we can say;

∠A + ∠B + ∠C = ∠D + ∠E + ∠F

∠A + ∠E + ∠F = ∠D + ∠E + ∠F [Since, ∠B = ∠E and ∠C = ∠F] ∠A = ∠D

Hence, in triangle ABC and DEF,

∠A = ∠D

AC = DF

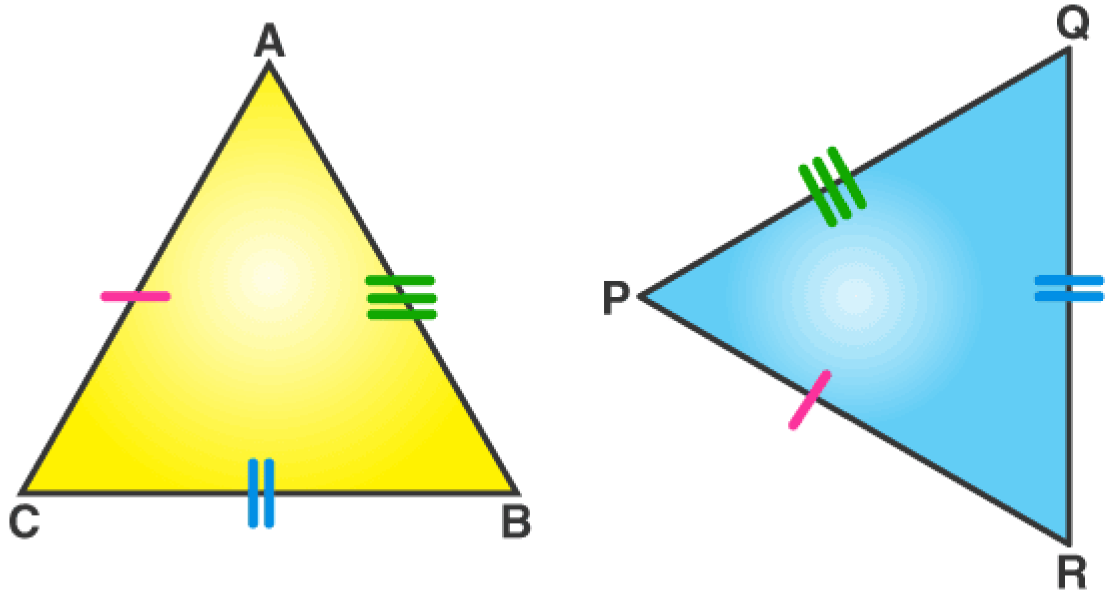
∠C = ∠F

Hence, by ASA congruency,

Δ ABC ≅ Δ DEF

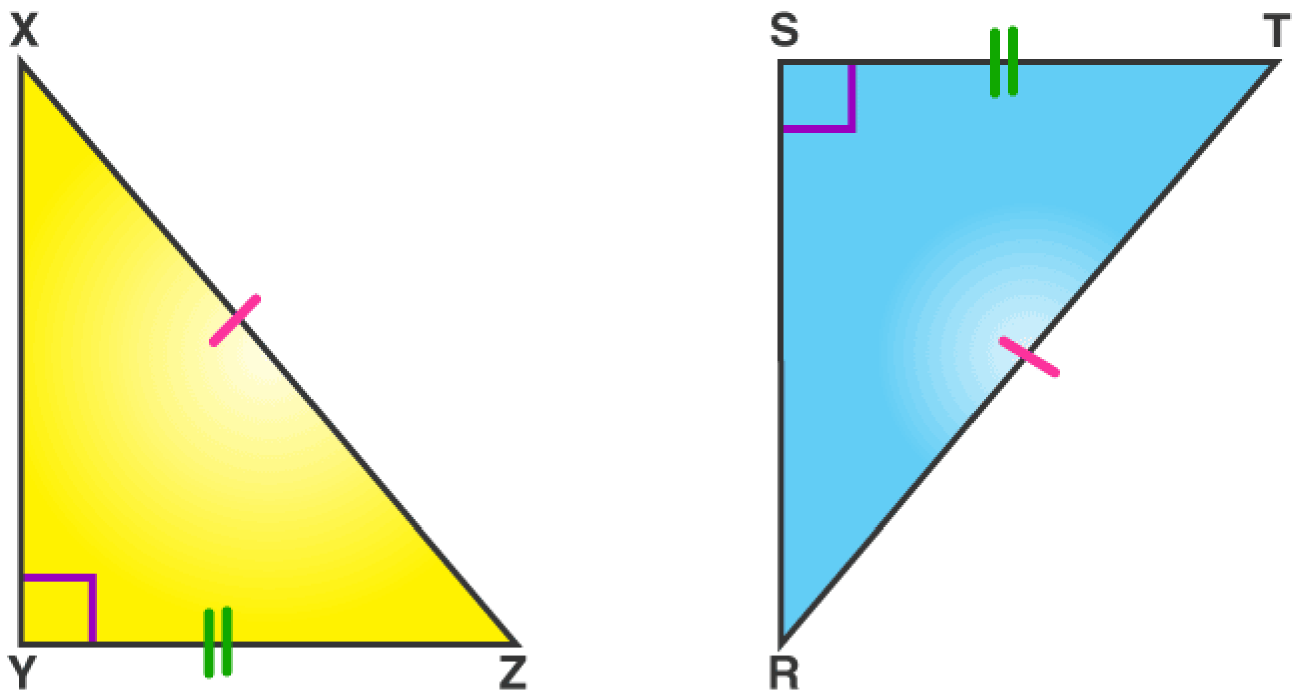
SSS (Side – Side – Side) congruent rule

If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.



RHS (Right Angle – Hypotenuse – Side) congruence rule

If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.



In above figure, hypotenuse XZ = RT and side YZ=ST, hence triangle XYZ ≅ triangle RST.

Congruence of Triangles

Congruence of triangles: Two triangles are said to be congruent if all three corresponding sides are equal and all the three corresponding angles are equal in measure. These triangles can be slides, rotated, flipped and turned to be looked identical. If repositioned, they coincide with each other. The symbol of congruence is' ≅'.

The corresponding sides and angles of congruent triangles are equal. There are basically four congruency rules that proves if two triangles are congruent. But it is necessary to find all six dimensions. Hence, the congruence of triangles can be evaluated by knowing only three values out of six. The meaning of congruence in Maths is when two figures are similar to each other based on their shape and size. Also, learn about Congruent Figures here.

Congruence is the term used to define an object and its mirror image. Two objects or shapes are said to be congruent if they superimpose on each other. Their shape and dimensions are the same. In the case of geometric figures, line segments with the same length are congruent and angles with the same measure are congruent.

CPCT is the term, we come across when we learn about the congruent triangle. Let's see the condition for triangles to be congruent with proof.

Congruent meaning in Maths

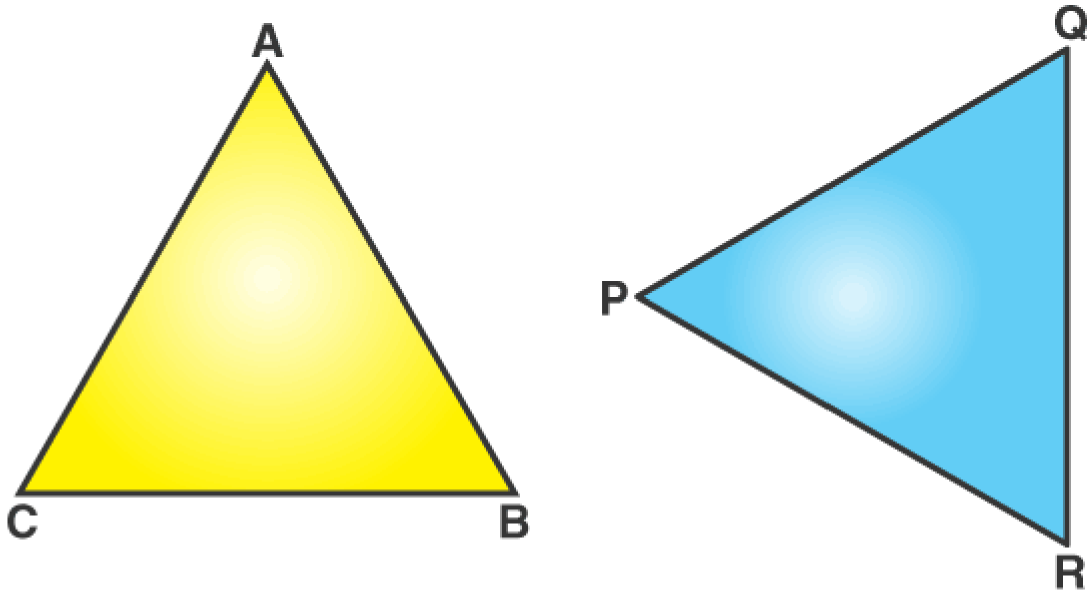
The meaning of congruent in Maths is addressed to those figures and shapes that can be repositioned or flipped to coincide with the other shapes. These shapes can be reflected to coincide with similar shapes.

Two shapes are congruent if they have the same shape and size. We can also say if two shapes are congruent, then the mirror image of one shape is same as the other.

Congruent Triangles

A polygon made of three line segments forming three angles is known as a Triangle.

Two triangles are said to be congruent if their sides have the same length and angles have same measure. Thus, two triangles can be superimposed side to side and angle to angle.



In the above figure, Δ ABC and Δ PQR are congruent triangles. This means,

Vertices: A and P, B and Q, and C and R are the same.

Sides: AB=PQ, QR= BC and AC=PR;

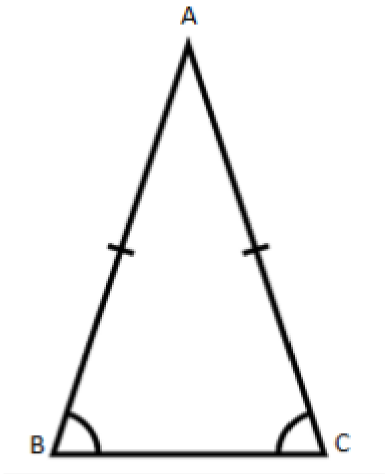
Angles: ∠A = ∠P, ∠B = ∠Q, and ∠C = ∠R.

Congruent triangles are triangles having corresponding sides and angles to be equal. Congruence is denoted by the symbol "≅". They have the same area and the same perimeter.

Isosceles triangle and its properties

- A triangle in which two sides are equal is called an **isosceles** triangle.
- Angles opposite to equal sides of an isosceles triangle are equal.
- The sides opposite to equal angles of a triangle are equal.

An isosceles triangle definition states it as a polygon that consists of two equal sides, two equal angles, three edges, three vertices and the sum of internal angles of a triangle equal to 1800. In this section, we will discuss the properties of isosceles triangle along with its definitions and its significance in Maths.



Apart from the isosceles triangle, there is a different classification of triangles depending upon the sides and angles, which have their own individual properties as well. Below is the list of types of triangles;

- Scalene Triangle
- Equilateral Triangle
- Acute angled Triangle
- Right angle Triangle
- Obtuse-angled Triangle

Isosceles triangle basically has two equal sides and angles opposite to these equal sides are also equal. Same like the Isosceles triangle, scalene and equilateral are also classified on the basis of their sides, whereas acute-angled, right-angled and obtuse-angled triangles are defined on the basis of angles. So before, discussing the properties of isosceles triangles, let us discuss first all the types of triangles.

Below are basic definitions of all types of triangles:

Scalene Triangle: A triangle which has all the sides and angles, unequal.

Equilateral Triangle: A triangle whose all the sides are equal and all the three angles are of 60^0.

Acute Angled Triangle: A triangle having all its angles less than right angle or 90^0.

Right Angled Triangle: A triangle having one of the three angles as right angle or 90^0.

Obtuse Angled Triangle: A triangle having one of the three angles as more than right angle or 90^0.

Isosceles Triangle Properties

An Isosceles Triangle has the following properties:

- Two sides are congruent to each other.
- The third side of an isosceles triangle which is unequal to the other two sides is called the base of the isosceles triangle.
- The two angles opposite to the equal sides are congruent to each other. That means it has two congruent base angles and this is called an isosceles triangle base angle theorem.
- The angle which is not congruent to the two congruent base angles is called an apex angle.
- The altitude from the apex of an isosceles triangle bisects the base into two equal parts and also bisects its apex angle into two equal angles.
- The altitude from the apex of an isosceles triangle divides the triangle into two congruent right-angled triangles.
- Area of Isosceles triangle = ½ × base × altitude
- Perimeter of Isosceles triangle = sum of all the three sides

Example: If an isosceles triangle has lengths of two equal sides as 5 cm and base as 4 cm and an altitude are drawn from the apex to the base of the triangle. Then find its area and perimeter.

Solution: Given the two equal sides are of 5 cm and base is 4 cm.

We know, the area of Isosceles triangle = ½ × base × altitude

Therefore, we have to first find out the value of altitude here.

The altitude from the apex divides the isosceles triangle into two equal right angles and bisects the base into two equal parts. Thus, by Pythagoras theorem,

$\text{Hypotenuse}^2 = \text{Base}^2 + \text{Perpendicular}^2$

Or Perpendicular = $\sqrt{Hypotenuse^2 - Base^2}$

∴ Altitude = $\sqrt{5^2 - 2^2}$

= $\sqrt{25 - 4}$

$= \sqrt{21}$

So, the area of Isosceles triangle = ½ × 4 × √21 = 2√21 cm2

Perimeter of Isosceles triangle = sum of all the sides of the triangle

= 5 + 4 + 5 cm

= 14 cm

Inequalities in a triangle

- If two sides of a triangle are unequal, the **angle opposite to the longer side is greater**.
- In any triangle, the **side opposite to greater (larger) angle is longer**.
- The **sum of any two sides** of a triangle is **greater than the third side**.
- The **difference between any two sides** of a triangle is **less than the third side**.

Relationship between unequal sides of the triangle and the angles opposite to it.

If 2 sides of a triangle are unequal, then the angle opposite to the longer side will be larger than the angle opposite to the shorter side.

Triangle inequality

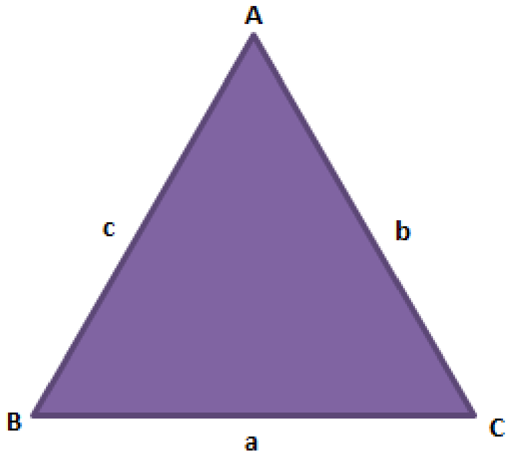
The sum of the lengths of any two sides of a triangle must be greater than the third side.

According to triangle inequality theorem, for any given triangle, the sum of two sides of a triangle is always greater than the third side. A polygon bounded by three line-segments is known as the Triangle. It is the smallest possible polygon. A triangle has three sides, three vertices, and three interior angles. The types of triangles are based on its angle measure and length of the sides. The inequality theorem is applicable for all types triangles such as equilateral, isosceles and scalene. Now let us learn this theorem in details with its proof.

Triangle Inequality Theorem Proof

The triangle inequality theorem describes the relationship between the three sides of a triangle. According to this theorem, for any triangle, the sum of lengths of two sides is always greater than the third side. In other words, this theorem specifies that the shortest distance between two distinct points is always a straight line.

Consider a ΔABC as shown below, with a, b and c as the side lengths.



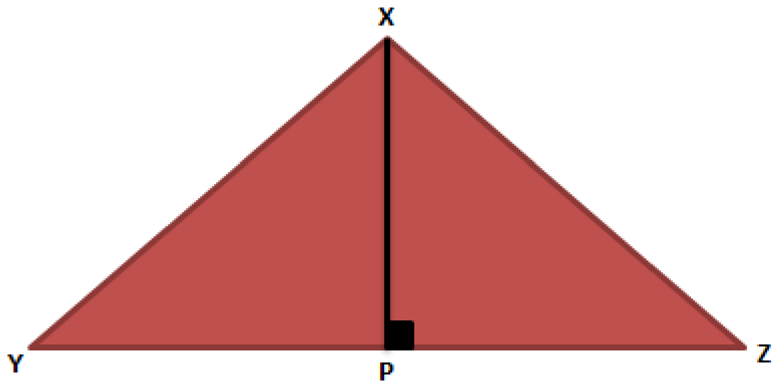
The triangle inequality theorem states that:

a < b + c,

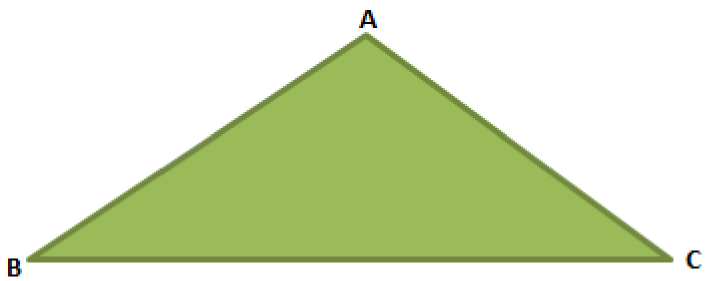
b < a + c,

c < a + b

In any triangle, the shortest distance from any vertex to the opposite side is the Perpendicular. In figure below, XP is the shortest line segment from vertex X to side YZ.

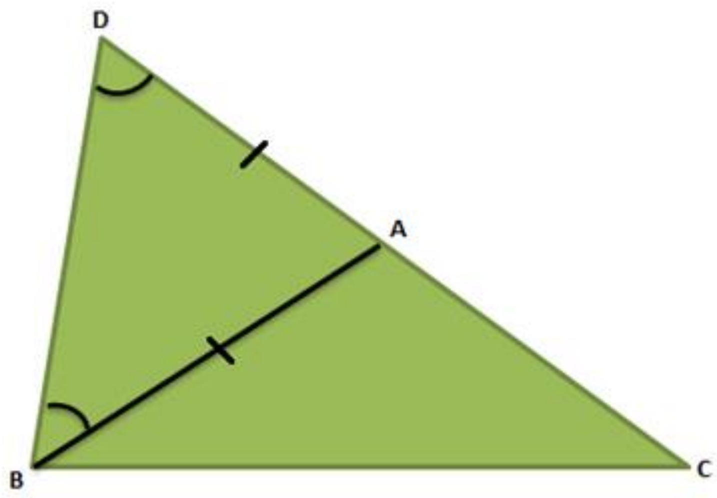


Let us prove the theorem now for a triangle ABC.



To Prove: |BC|< |AB| + |AC|

Construction: Consider a ΔABC. Extend the side AC to a point D such that AD = AB as shown in the fig. below.



Proof of triangle inequality theorem

CHAPTER : 7 TRIANGLES

Triangles

Triangle

closed figure formed by three straight lines

It has three - sides, angles and vertices each

ΔABC

Congruent

If any three parameters of given triangles are same, the triangles will be congruent.

$\Delta ABC \cong \Delta DEF$

Congruence rule

Rule	Statement	Figure
1. SAS	Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.	In ΔAOD and ΔCOB $CO = OD$ $\angle COB = \angle AOD$ $OB = OA$ $\therefore\ \Delta AOD \cong \Delta COB$
2. ASA	Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle.	In ΔABC and ΔDEF $\angle B = \angle E$ $BC = EF$ $\angle C = \angle F$ $\therefore\ \Delta ABC \cong \Delta DEF$
3. AAS	Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.	Given AB || CD In ΔAOB and ΔCOD $\angle ABO = \angle DCO$ $\angle AOB = \angle DOC$ $OA = OD$ $\therefore\ \Delta AOB \cong \Delta COD$
4. SSS	If three sides of one triangle are equal to the three sides of another triangle, then two triangles are congruent.	In ΔABC and ΔDEF $AC = DF$ $AB = DE$ $BC = FE$ $\therefore\ \Delta ABC \cong \Delta DEF$
5. RHS	If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.	In ΔABC and ΔDEF $AC = DF = 5cm$ $BC = FE = 4cm$ $AB = \sqrt{AC^2 - BC^2} = \sqrt{5^2 - 4^2} = 3$ $DE = \sqrt{DF^2 - EF^2} = \sqrt{5^2 - 4^2} = 3$ $\therefore\ AB = DE$ Hence $\Delta ABC \cong \Delta DEF$

Properties

Statement	Figure
Angles opposite to equal side of an isosceles triangle are equal	$AB = AD$ $\angle B = \angle C$
The sides opposite to equal angles of a triangle are equal	$\angle BAC = \angle CAD$ $\angle ADB = \angle ADC$ $\Delta ABD \cong \Delta ACD$ (ASA rule) Hence, $AB = AC$

Inequalities

Statement	Figure
In any triangle, the angle opposite to the longer side is larger.	AC is the longest side $\therefore\ \angle B$ is largest
In any triangle, the side opposite to the larger (greater) angle is longer.	if $\angle B$ is the largest $\therefore$ AC is longest
The sum of any two sides of a triangle is greater than the third side.	In ΔABC $AB + AC > BC$ $AB + BC > AC$ $AB + BC > AB$
Difference of any two sides of a triangle is less than the third side.	In ΔABC $AB - BC < CA$ $AB - AC < BC$ $AC - BC < AB$

Important Questions

Multiple Choice Questions-

Question 1. △ABC = △PQR, then which of the following is true?

(a) CB = QP

(b) CA = RP

(c) AC = RQ

(d) AB = RP

Question 2. In △ABC and △DEF, AB = DE and ∠A = ∠D. Then two triangles will be congruent by SA axiom if:

(a) BC = EF

(b) AC = EF

(c) AC = DE

(d) BC = DE

Question 3. In a right triangle, the longest side is:

(a) Perpendicular

(b) Hypotenuse

(c) Base

(d) None of the above

Question 4. In △ABC, if ∠A = 45° and ∠B = 70°, then the shortest and the longest sides of the triangle are respectively,

(a) BC, AB

(a) AB, AC

(c) AB, BC

(d) BC, AC

Question 5. If the altitudes from vertices of a triangle to the opposite sides are equal, then the triangles is

(a) Scalene

(b) Isosceles

(c) Equilateral

(d) Right-angled

Question 6. D is a Point on the Side BC of a △ABC such that AD bisects ∠BAC then:

(a) BD = CD

(b) CD > CA

(c) BD > BA

(d) BA > BD

Question 7. If ΔABC ≅ ΔPQR then which of the following is true:

(a) CA = RP

(b) AB = RP

(c) AC = RQ

(d) CB = QP

Question 8. If two triangles ABC and PQR are congruent under the correspondence A ↔ P, B ↔ Q, and C ↔ R, then symbolically, it is expressed as

(a) ΔABC ≅ ΔPQR

(b) ΔABC = ΔPQR

(c) ΔABC and ΔPQR are scalene triangles

(d) ΔABC and ΔPQR are isosceles triangles

Question 9. If the bisector of the angle A of an △ABC is perpendicular to the base BC of the triangle then the triangle ABC is:

(a) Obtuse Angled

(b) Isosceles

(c) Scalene

(d) Equilateral

Question 10.

If AB = QR, BC=RP and CA = QP, then which of the following holds?

(a) △BCA ≅ △PQR

(b) △ABC ≅ △PQR

(c) △CBA ≅ △PQR

(d) △CAB ≅ △PQR

Very Short:

1. Find the measure of each exterior angle of an equilateral triangle.

2. If in ΔABC, ∠A = ∠B + ∠C, then write the shape of the given triangle.

3. In ΔPQR, PQ = QR and ∠R = 50°, then find the measure of ∠Q.

4. If ΔSKY ≅ ΔMON by SSS congruence rule, then write three equalities of

corresponding angles.

5. Is ΔABC possible, if AB = 6 cm, BC = 4 cm and AC = 1.5cm?

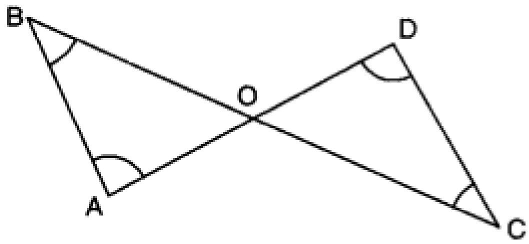
6. In ΔMNO, if ∠N = 90°, then write the longest side.

7. In ΔABC, if AB = AC and ∠B = 70°, find ∠A.

8. In ΔABC, if AD is a median, then show that AB + AC > 2AD.

Short Questions:

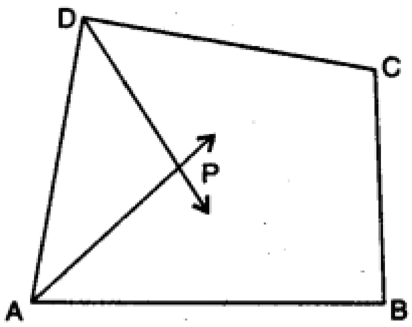
1. In the given figure, AD = BC and BD = AC, prove that ∠DAB = ∠CBA.
2. In the given figure, ΔABD and ABCD are isosceles triangles on the same base BD. Prove that ∠ABC = ∠ADC.
3. In the given figure, if ∠1 = ∠2 and ∠3 = ∠4, then prove that BC = CD.
4. In the given figure, ∠B < ∠A and ∠C < ∠D. Show that AD < BC.



5. In the given figure, AC > AB and D is a point on AC such that AB = AD. Show that BC > CD.
6. In a triangle ABC, D is the mid-point of side AC such that BD $= \frac{1}{2}$ AC Show that ∠ABC is a right angle.

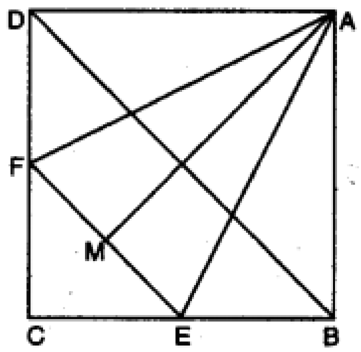
Long Questions:

1. In the given figure, AP and DP are bisectors of two adjacent angles A and D of quadrilateral ABCD. Prove that 2 ∠APD = ∠B + 2C

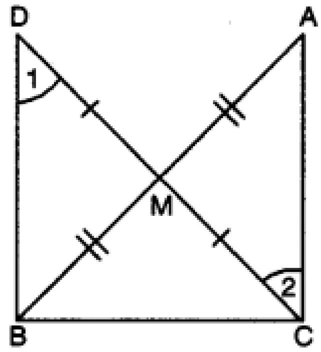


2. In figure, ABCD is a square and EF is parallel to diagonal BD and EM = FM. Prove that

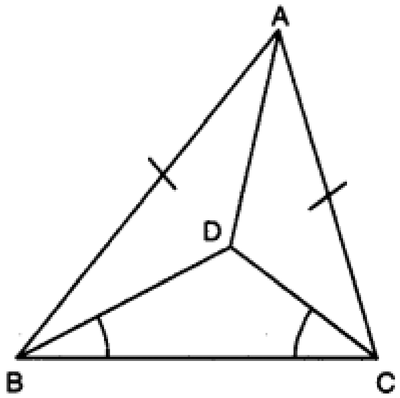
   (i) DF = BE (i) AM bisects ∠BAD.



3. In right triangle ABC, right-angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see fig.). Show that : (i) ΔAMC ≅ ΔBMD (ii) ∠DBC = 90° (ii) ΔDBC ≅ ΔACB (iv) $CM = \frac{1}{2}\ AB$



4. In figure, ABC is an isosceles triangle with AB = AC. D is a point in the interior of ΔABC such that ∠BCD = ∠CBD. Prove that AD bisects ∠BAC of ΔABC.



5. Prove that two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle.

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: If we draw two triangles with angles 30°, 70° and 80° and the length of the sides of one triangle be different than that of the corresponding sides of the other triangle then two triangles are not congruent.

Reason: If two triangles are constructed which have all corresponding angles equal but have unequal corresponding sides, then two triangles cannot be congruent to each other.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

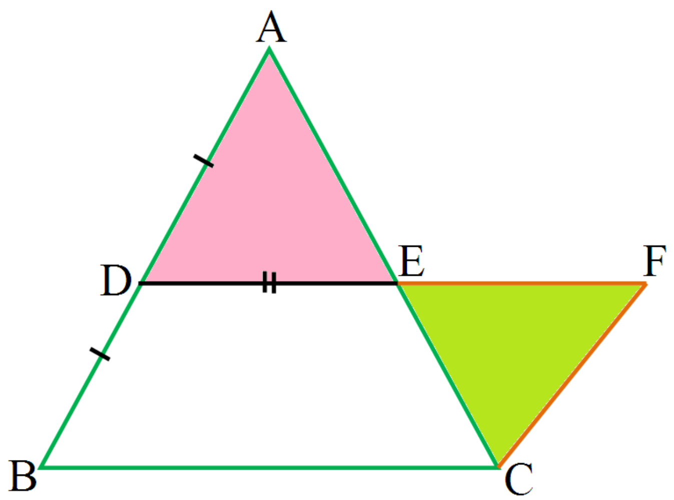
d) Assertion is wrong statement but reason is correct statement.

Assertion: If the bisector of the vertical angle of a triangle bisects the base of the triangle, then the triangle is equilateral.

Reason: If three sides of one triangle are equal to three of the other triangle, then the two triangles are congruent.

Case Study Questions-

1. Read the Source/ Text given below and answer these questions:



Hareesh and Deep were trying to prove a theorem. For this they did the following:

i. Drew a triangle ABC.

ii. D and E are found as the mid points of AB and AC.

iii. DE was joined and DE was extended to F so DE = EF.

iv. FC was joined.

Answer the following questions:

i. △ADE and △EFC are congruent by which criteria?

a. SSS

b. RHS

c. SAS

d. ASA

ii. ∠EFC is equal to which angle?

a. ∠DAE

b. ∠ADE

c. ∠AED

d. ∠B

iii. ∠ECF is equal to which angle?

a. ∠DAE

b. ∠ADE

c. ∠AED

d. ∠B

iv. CF is equal to which of the following?

a. BD

b. CE

c. AE

d. EF

v. CF is parallel to which of the following?

a. AE

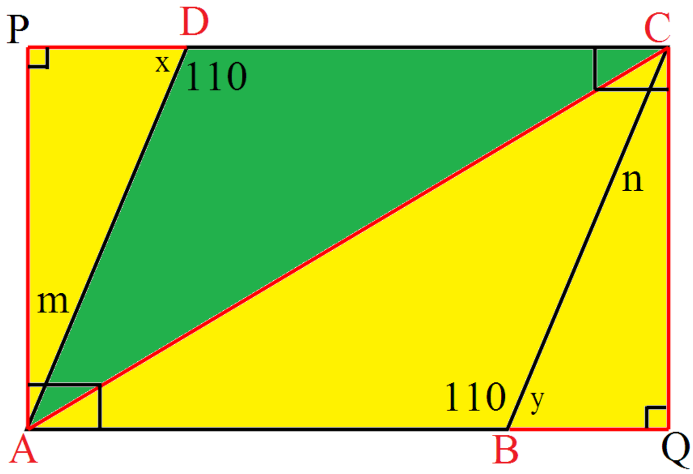
b. CE

c. BD

d. EF

2. Read the Source/ Text given below and answer these questions:

In the middle of the city, there was a park ABCD in the form of a parallelogram form so that AB = CD, AB || CD and AD = BC, AD || BC Municipality converted this park into a rectangular form by adding land in the form of △APD and △BCQ. Both the triangular shape of land were covered by planting flower plants.



Answer the following questions:

i. What is the value of ∠x?

a. 110°

b. 70°

c. 90°

d. 100°

ii. △APD and △BCQ are congruent by which criteria?

a. SSS

b. SAS

c. ASA

d. RHS

iii. PD is equal to which side?

a. DC

b. AB

c. BC

d. BQ

iv. △ABC and △ACD are congruent by which criteria?

a. SSS

b. SAS

c. ASA

d. RHS

v. What is the value of ∠m?

a. 110°

b. 70°

c. 90°

d. 20°

Answer Key:

MCQ:

1. (b) CA = RP
2. (c) AC = DE
3. (b) Hypotenuse
4. (d) BC,AC
5. (b) Isosceles
6. (d) BA > BD
7. (a) CA = RP
8. (a) $\Delta ABC \cong \Delta PQR$
9. (b) Isosceles
10. (d) $\triangle CAB \cong \triangle PQR$

Very Short Answer:

1. We know that each interior angle of an equilateral triangle is 60°.

   ∴ Each exterior angle = $180° – 60° = 120°$

2. Here, $\angle A = \angle B + \angle C$

   And in ΔABC, by angle sum property, we have

   $\angle A + \angle B + C = 180°$

   $\Rightarrow \angle A + \angle A = 180°$

   $\Rightarrow 2\angle A = 180°$

   $\Rightarrow \angle A = 90°$

   Hence, the given triangle is a right triangle.

3. Here, in ΔPQR, PQ = QR

   $\Rightarrow \angle R = \angle P = 50°$ (given)

   Now, $\angle P + \angle Q + \angle R = 180°$

   $50° + \angle Q + 50° = 180°$

   $\Rightarrow \angle Q = 180° – 50° – 50°$

   $= 80°$

4. Since $\Delta SKY \cong \Delta MON$ by SSS congruence rule, then three equalities of

corresponding angles

are $\angle S = \angle M$, $\angle K = \angle O$ and $\angle Y = \angle N$.

5. Since $4 + 1.5 = 5.5 \neq 6$

Thus, triangle is not possible.

6. We know that, side opposite to the largest angle is longest.

$\therefore$ Longest side = MO.

7. Here, in ΔABC $AB = AC$ $\angle C = \angle B$ [$\angle$s opp. to equal sides of a Δ)

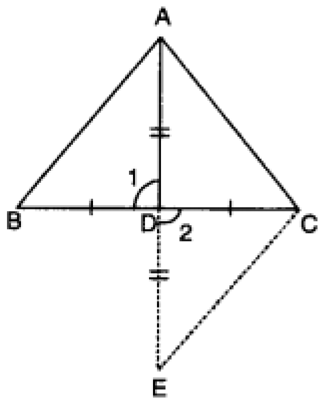
Now, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 70° + 70° = 180°$

[$\because \angle B = 70°$]

$\Rightarrow \angle A = 180° - 70° - 70° = 40°$

8.



Produce AD to E, such that AD = DE.

In ΔADB and ΔEDC, we have

$BD = CD$, $AD = DE$ and $\angle 1 = \angle 2$

$\Delta ADB \cong \Delta EDC$

$AB = CE$

Now, in ΔAEC, we have

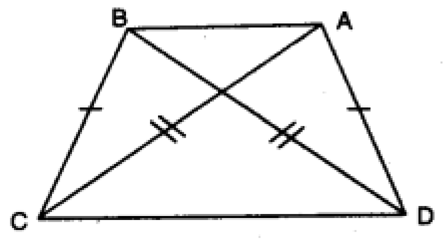
$AC + CE > AE$

$AC + AB > AD + DE$

$AB + AC > 2AD$ [$\because AD = DE$]

Short Answer:

Ans: 1.



In ΔDAB and ΔCBA, we have

AD = BC [given]

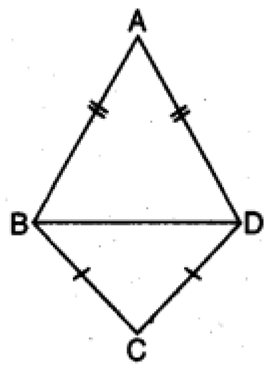
BD = AC [given]

AB = AB [common]

∴ ΔDAB ≅ ΔCBA [by SSS congruence axiom]

Thus, ∠DAB =∠CBA [c.p.c.t.]

Ans: 2.



In ΔABD, we have

AB = AD (given)

∠ABD = ∠ADB [angles opposite to equal sides are equal] ...(i)

In ΔBCD, we have

CB = CD

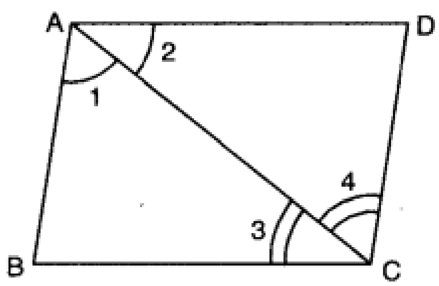
⇒ ∠CBD = ∠CDB [angles opposite to equal sides are equal] ... (ii)

Adding (i) and (ii), we have

∠ABD + ∠CBD = ∠ADB + ∠CDB

⇒ ∠ABC = ∠ADC

Ans: 3.



In ΔABC and ACDA, we have

∠1 = ∠2 (given)

AC = AC [common]

∠3 = ∠4 [given]

So, by using ASA congruence axiom

ΔABC ≅ ΔCDA

Since corresponding parts of congruent triangles are equal

∴ BC = CD

Ans: 4. In ΔABC and ACDA, we have

∠1 = ∠2 (given)

AC = AC [common]

∠3 = ∠4 [given]

So, by using ASA congruence axiom

ΔABC ≅ ΔCDA

Since corresponding parts of congruent triangles are equal

∴ BC = CD

Ans: 5. Here, in ΔABD, AB = AD

∠ABD = ∠ADB

[∠s opp. to equal sides of a Δ]

In ΔBAD

ext. ∠BDC = ∠BAD + ∠ABD

⇒ ∠BDC > ∠ABD .... (ii)

Also, in ΔBDC.

ext. ∠ADB > ∠CBD ...(iii)

From (ii) and (iii), we have

∠BDC > CD [∵ sides opp. to greater angle is larger]

Ans: 6. Here, in ΔABC, D is the mid-point of AC.

⇒ AD = CD $= \frac{1}{2}$ AC ....(i)

Also, BD $= \frac{1}{2}$ AC .... (ii) [Given]

From (i) and (ii), we obtain

AD = BD and CD = BD

⇒ ∠2 = ∠4 and ∠1 = ∠3 .....(iii)

In ΔABC, we have

∠ABC + ∠ACB + ∠CAB = 180°

⇒ ∠1 + ∠2 + ∠3 + ∠4 = 180°

⇒ ∠1 + ∠2 + ∠1 + ∠2 = 180° [using (iii)]

⇒ 2(∠1 + ∠2) = 180°

⇒ ∠1 + ∠2 = 90°

Hence, ∠ABC = 90°

Long Answer:

Ans: 1. Here, AP and DP are angle bisectors of ∠A and ∠D

∴ $\angle DAP = \frac{1}{2}\angle DAB$ and $\angle ADP = \frac{1}{2}\angle ADC$ ......(i)

In ΔAPD, ∠APD + ∠DAP + ∠ADP = 180°

⇒ $\angle APD + \frac{1}{2}\angle DAB + \frac{1}{2}\angle ADC = 180°$

⇒ $\angle APD = 180° - \frac{1}{2}(\angle DAB + \angle ADC)$

⇒ 2∠APD = 360° – (∠DAB + ∠ADC) ......(ii)

Also, ∠A + ∠B + C + ∠D = 360°

∠B + 2C = 360° – (∠A + ∠D)

∠B + C = 360° – (∠DAB + ∠ADC) ......(iii)

From (ii) and (iii), we obtain

2∠APD = ∠B + ∠C

Ans: 2. (i) EF | | BD = ∠1 = ∠2 and ∠3 = ∠4 [corresponding ∠s]

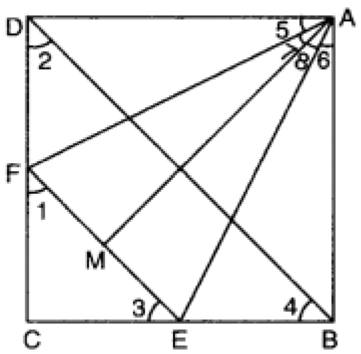
Also, ∠2 = ∠4

⇒ ∠1 = ∠3

⇒ CE = CF (sides opp. to equals ∠s of a Δ]

∴ DF = BE

[∵ BC – CE = CD – CF)



(ii) In ΔADF and ΔABE, we have

AD = AB [sides of a square]

DF = BE [proved above]

∠D = ∠B = 90°

⇒ ΔADF ≅ ΔABE [by SAS congruence axiom]

⇒ AF = AE and ∠5 = ∠6 … (i) [c.p.c.t.]

In ΔAMF and ΔAME

AF = AE [proved above]

AM = AM [common]

FM = EM (given)

∴ ΔAMF ≅ ΔAME [by SSS congruence axiom]

∴ ∠7 = ∠8 …(ii) [c.p.c.t.]

Adding (i) and (ii), we have

∠5 + ∠7 = ∠6 + ∠8

∠DAM = ∠BAM

∴ AM bisects ∠BAD.

Ans: 3. Given: ΔACB in which 4C = 90° and M is the mid-point of AB.

To Prove:

(i) ΔAMC ≅ ΔBMD

(ii) ∠DBC = 90°

(iii) ΔDBC ≅ ΔACB

(iv) CM $= \frac{1}{2}\ \mathrm{AB}$

Proof: Consider ΔAMC and ΔBMD,

we have AM = BM [given]

CM = DM [by construction]

∠AMC = ∠BMD [vertically opposite angles]

∴ ΔAMC ≅ ΔBMD [by SAS congruence axiom]

⇒ AC = DB …(i) [by c.p.c.t.]

and ∠1 = ∠2 [by c.p.c.t.]

But ∠1 and ∠2 are alternate angles.

⇒ BD || CA

Now, BD || CA and BC is transversal.

∴ ∠ACB + ∠CBD = 180°

⇒ 90° + CBD = 180°

⇒ ∠CBD = 90°

In ΔDBC and ΔACB,

we have CB = BC [common]

DB = AC [using (i)]

∠CBD = ∠BCA

∴ ΔDBC ≅ ΔACB

⇒ DC = AB

$\Rightarrow \frac{1}{2}\ \mathrm{AB} = \frac{1}{2}\ \mathrm{DC}$

$\Rightarrow \frac{1}{2}\ \mathrm{AB} = \mathrm{CM\ Or\ CM} = \frac{1}{2}\ \mathrm{AB}\ (\because \mathrm{CM} = \frac{1}{2}\ \mathrm{DC})$

Ans: 4. In ΔBDC, we have ∠DBC = ∠DCB (given).

⇒ CD = BD (sides opp. to equal ∠s of ΔDBC)

Now, in ΔABD and ΔACD,

we have AB = AC [given]

BD = CD [proved above]

AD = AD [common]

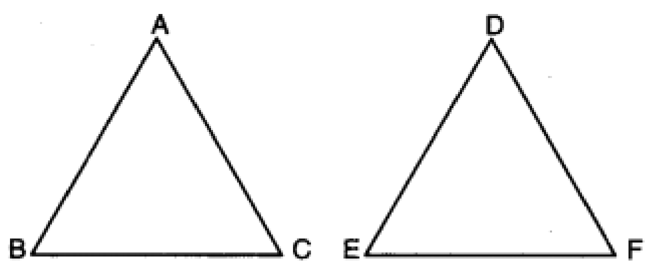
∴ By using SSS congruence axiom, we obtain

ΔABD ≅ ΔACD

⇒ ∠BAD = ∠CAD [c.p.ç.t.]

Hence, AD bisects ∠BAC of ΔABC.

Ans: 5.



Given: Two As ABC and DEF in which

∠B = ∠E,

∠C = ∠F and BC = EF

To Prove: ΔABC = ΔDEF

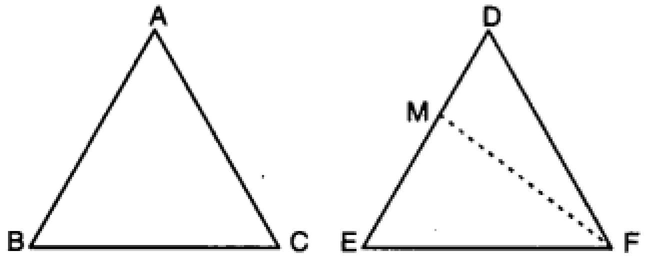
Proof: We have three possibilities

Case I. If AB = DE,

we have AB = DE,

∠B = ∠E and BC = EF.

So, by SAS congruence axiom, we have ΔABC ≅ ΔDEF



Case II. If AB < ED, then take a point Mon ED

such that EM = AB.

Join MF.

Now, in ΔABC and ΔMEF,

we have

AB = ME, ∠B = ∠E and BC = EF.

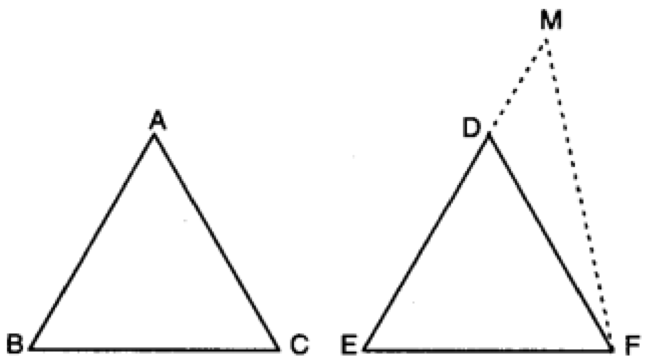
So, by SAS congruence axiom,

we have ΔABC ≅ ΔMEF

⇒ ∠ACB = ∠MFE

But ∠ACB = ∠DFE

∴ ∠MFE = ∠DFE



Which is possible only when FM coincides with B FD i.e., M coincides with D.

Thus, AB = DE

∴ In ΔABC and ΔDEF, we have

AB = DE,

∠B = ∠E and BC = EF

So, by SAS congruence axiom, we have

$\Delta ABC \cong \Delta DEF$

Case III. When AB > ED

Take a point M on ED produced

such that EM = AB.

Join MF

Proceeding as in Case II, we can prove that

$\Delta ABC = \Delta DEF$

Hence, in all cases, we have

$\Delta ABC = \Delta DEF$.

Assertion and Reason Answers-

1. b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

2. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Case Study Answers-

1.

(i)	(c)	SAS
(ii)	(b)	∠ADE∠ADE
(iii)	(a)	∠DAE∠DAE
(iv)	(a)	BD
(v)	(c)	BD

2.

(i)	(b)	70°
(ii)	(c)	ASA
(iii)	(d)	BQ
(iv)	(a)	SSS
(v)	(d)	20°