MATHEMATICS

Chapter 13: Surface Areas and Volumes

Surface Areas and Volumes

1. A cuboid is a solid bounded by six rectangular plane regions. It has length, width and height.

2. A cuboid whose all edges are equal is called a cube.

3. A cylinder is a closed solid that has two parallel (usually circular) bases connected by a curved surface.

4. A cone is a solid that has a circular base and a single vertex.

5. A sphere is a perfectly round geometrical object in three-dimensional space, such as the shape of a round ball.

6. A hemisphere is half of a sphere.

7. Surface area of a solid is the sum of the areas of all its faces.

8. The total surface area of any object will be greater than its lateral surface area.

9. In case of a room, lateral surface area means the area of the four walls of the room, whereas total surface area means the area of four walls plus the area of the floor and the ceiling.

10. Volume is the space occupied by an object.

11. The unit of measurement of both volume and capacity is cubic unit such as cubic feet, cubic cm. cubic m etc.

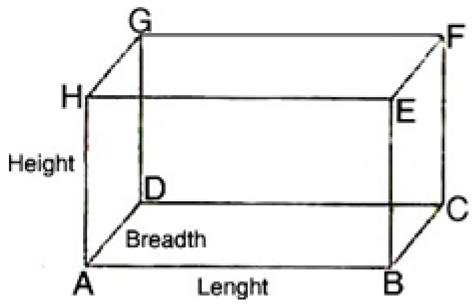
12. If l, b, h denote respectively the length, breadth and height of a **cuboid**, then:

Lateral surface area or Area of four walls = $2(\ell + b)h$

Total surface area = $2(\ell b + bh + h\ell)$

Volume = $\ell \times b \times h$

Diagonal of a cuboid $= \sqrt{l^2 + b^2 + h^2}$



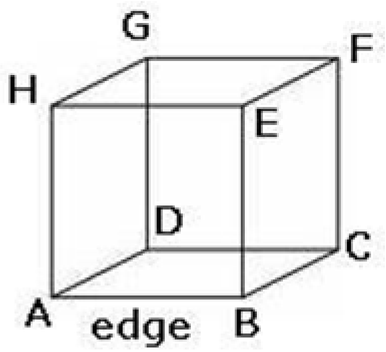
13. If the length of each edge of a **cube** is 'a' units, then:

Lateral surface area = 4 x $(\text{edge})^2$

Total surface area = 6 x $(\text{edge})^2$

Volume = $(\text{edge})^3$

Diagonal of a cube $= \sqrt{3}x$ edge

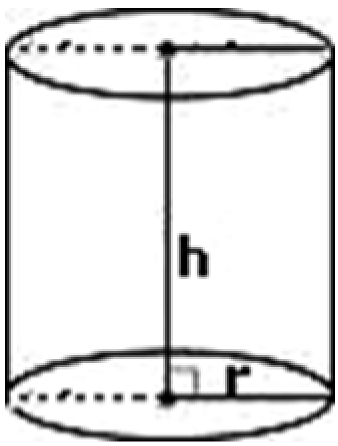


14. If r and h respectively denote the radius of the base and the height of a **right circular cylinder**, then: Area of each end or Base area $= \pi r^2$

Area of curved surface or lateral surface area = perimeter of the base x height = $2\pi rh$

Total surface area (including both ends) = $2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$

Volume = Area of the base × height = $\pi r^2 h$



15. If R and r respectively denote the external and internal radii of a **right circular hollow cylinder** and h denotes its height, then:

Area of each circular base = $\pi R^2 - \pi r^2$

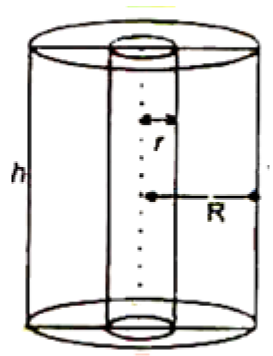
Area of curved surface = $2\pi(R + r)h$

Total surface area = (External surface) + (Internal surface)

$= (2\pi Rh + 2\pi rh) + 2\,(\pi R^2 - \pi r^2)$

Volume of = (External volume) – (Internal volume)

$= (\pi R^2 h - \pi r^2 h) = \pi h\,(R^2 - r^2)$

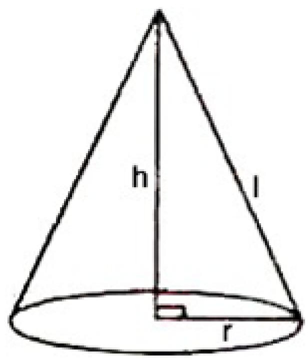


16. If r, h and l respectively denote the radius, height and slant height of a **right circular cone**, then: Slant height $(l) = \sqrt{h^2 + r^2}$

Area of curved surface $\pi r l = \pi r\sqrt{h^2 + r^2}$

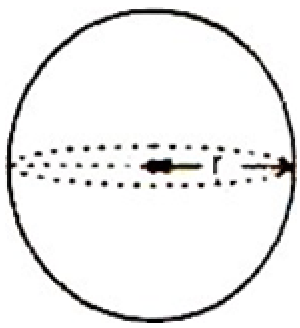
Total surface area = Area of curved surface + Area of base = $\pi r\ell + \pi r^2 = \pi r(\ell + r)$

Volume $= \frac{1}{3}\pi r^2 h$



17. If r is the radius of a **sphere**, then: Surface area = $4\pi r^2$

Volume $= \frac{4}{3}\pi r^3$



18. If r is the radius of a **hemisphere**, then:

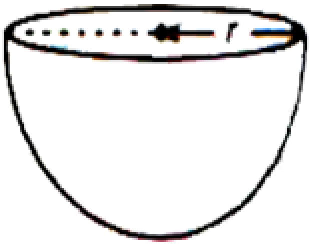
Area of curved surface = $2\pi r^2$

Total surface Area = Area of curved surface + Area of base

$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

Volume $= \frac{2}{3}\pi r^3$



19. Volume of water flown in the tank in one hour = (area of cross section of the aperture) × (speed in meters per hour)

20. When an object of certain volume is recast into a cylinder, the volume of the cylinder formed will always be equal to the volume of the original object.

21. The solids having the same curved surface do not necessarily occupy the same volume.

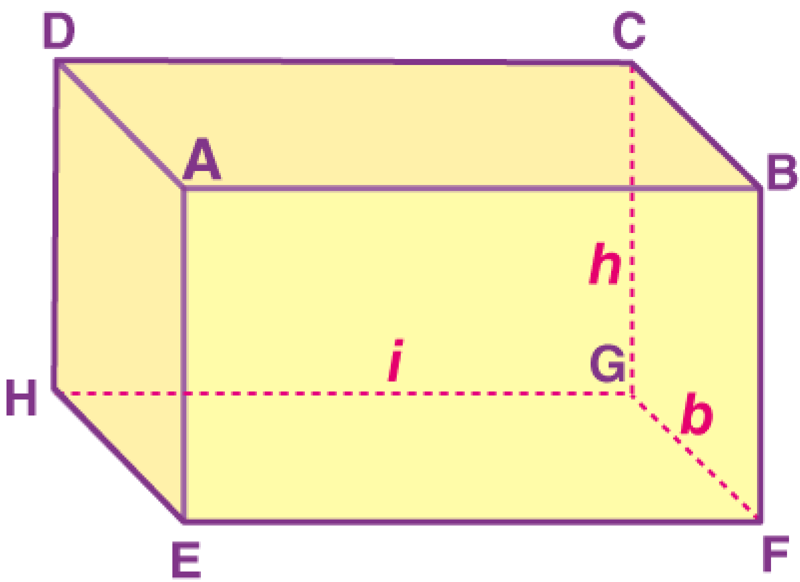
22. When an object is dropped into a liquid, the volume of the displaced liquid is equal to the volume of the object that is dipped.

23. Of all the solids having a given volume, the sphere is the one with the smallest surface area. Of all solids having a given surface area, the sphere is the one having the greatest volume.

Cuboid

A cuboid is a three-dimensional Shape. The cuboid is made from six rectangular faces, which are placed at right angles. The total surface area of a cuboid is equal to the sum of the areas of its six rectangular faces.

Total Surface Area of a Cuboid



Consider a cuboid whose length is "l" cm, breadth is b cm and height h cm.

Area of face ABCD = Area of Face EFGH = (l × b) cm^2

Area of face AEHD = Area of face BFGC = (b × h) cm^2

Area of face ABFE = Area of face DHGC = (l × h) cm^2

Total surface area (TSA) of cuboid = Sum of the areas of all its six faces

= 2(l × b) + 2(b × h) + 2(l × h)

TSA (cuboid)= 2(lb + bh +lh)

Lateral Surface Area of a Cuboid

Lateral surface area (LSA) is the area of all the sides apart from the top and bottom faces.

The lateral surface area of the cuboid

= Area of face AEHD + Area of face BFGC + Area of face ABFE + Area of face DHGC

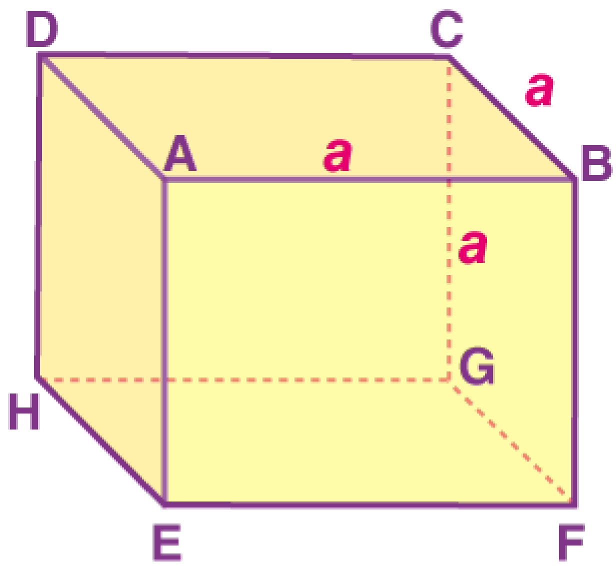
= 2(b × h) + 2(l × h)

LSA (cuboid) = 2h (l + b)

Cube

A cuboid whose length, breadth and height are all equal, is called a cube. It is a three-dimensional shape bounded by six equal squares. It has 12 edges and 8 vertices.

Total Surface Area of a cube



For cube, length = breadth = height

Suppose the length of an edge = a

Total surface area (TSA) of the cube = 2(a × a + a × a + a × a)

TSA (cube) = $2 \times (3a^2) = 6a^2$

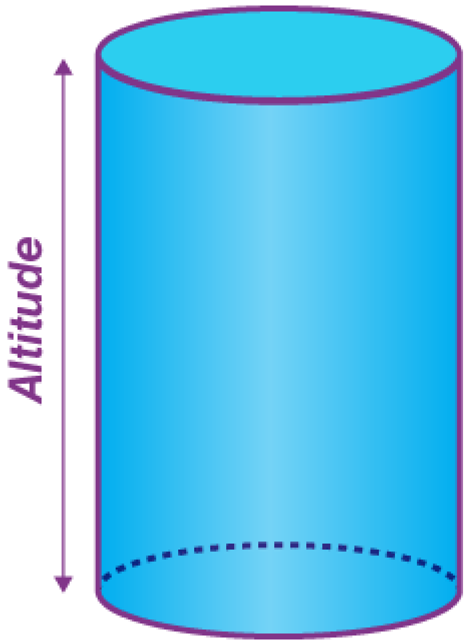
Lateral Surface area of a cube

Lateral surface area (LSA) is the area of all the sides apart from the top and bottom faces.

Lateral surface area of cube = $2(a \times a + a \times a) = 4a^2$

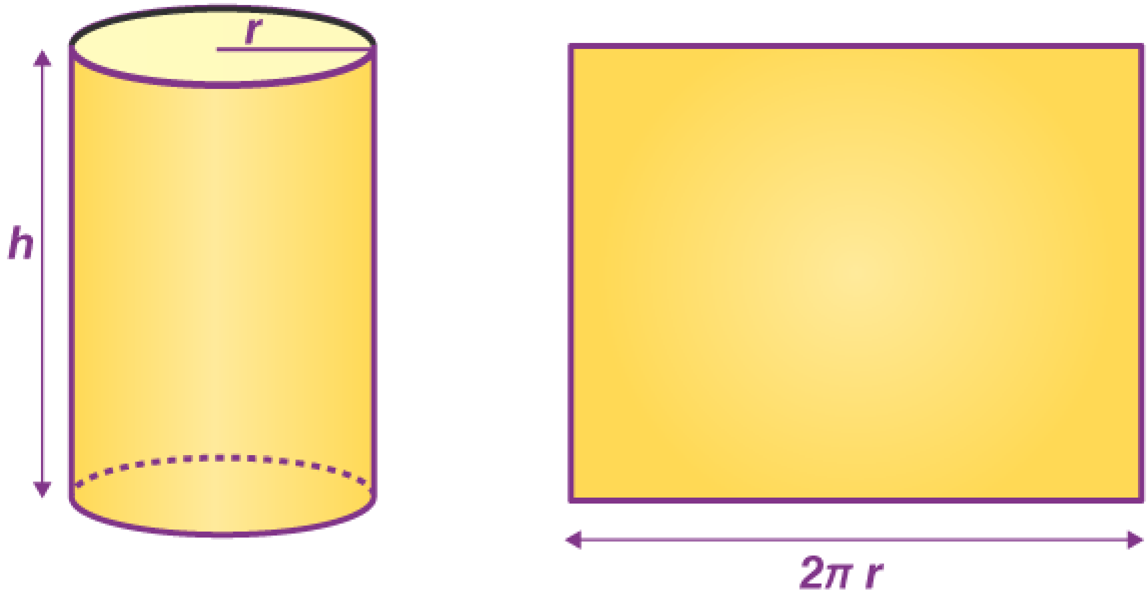
Right Circular Cylinder

A right circular cylinder is a closed solid that has two parallel circular bases connected by a curved surface in which the two bases are exactly over each other and the axis is at right angles to the base.



Curved Surface area of a right circular cylinder

Take a cylinder of base radius r and height h units. The curved surface of this cylinder, if opened along the diameter (d = 2r) of the circular base will be transformed into a rectangle of length 2πr and height h units. Thus,



Curved surface area (CSA) of a cylinder of base radius r and height h = $2\pi \times r \times h$

Total surface area of a right circular cylinder

Total surface area (TSA) of a cylinder of base radius r and height h = $2\pi \times r \times h$ + area of two circular bases

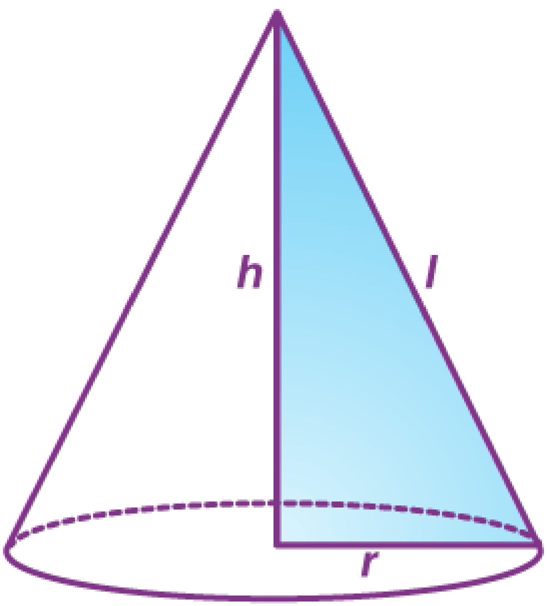
$\Rightarrow$ TSA = $2\pi \times r \times h + 2 \times \pi r^2$

$\Rightarrow$ TSA = $2\pi r\,(h + r)$

Right Circular Cone

A right circular cone is a circular cone whose axis is perpendicular to its base.

Relation between slant height and height of a right circular cone



The relationship between slant height(l) and height(h) of a right circular cone is:

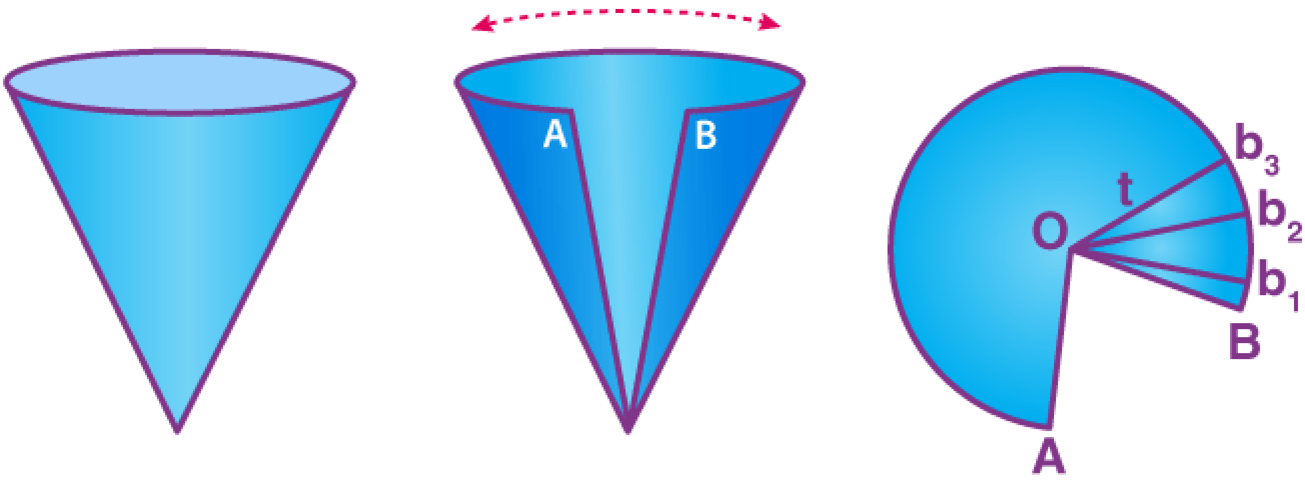
$l^2 = h^2 + r^2$ (Using Pythagoras Theorem)

Where r is the radius of the base of the cone.

Curved Surface Area of a Right Circular Cone

Consider a right circular cone with slant length l and radius r.

If a perpendicular cut is made from a point on the circumference of the base to the vertex and the cone is opened up, a sector of a circle with radius l is produced as shown in the figure below:



Label A and B and corresponding b_1, b_2 ...b_n at equal intervals, with O as the common vertex. The Curved surface area (CSA) of the cone will be the sum of areas of the small triangles: $1/2 \times (b_1 + b_2 \ldots\ldots b_n) \times l$

(b1 + b2...bn) is also equal to the circumference of base = $2\pi r$

CSA of right circular cone = $(1/2) \times (2\pi r) \times l = \pi rl$ (On substituting the values)

Total Surface Area of a Right Circular Cone

Total surface area (TSA) = Curved surface area(CSA) + area of base = $\pi rl + \pi r^2 = \pi r(l + r)$

Sphere

A sphere is a closed three-dimensional solid figure, where all the points on the surface of the sphere are equidistant from the common fixed point called "centre". The equidistant is called the "radius".

Surface area of a Sphere

The surface area of a sphere of radius r = 4 times the area of a circle of radius r = $4 \times (\pi r^2)$

For a sphere Curved surface area (CSA) = Total Surface area (TSA) = $4\pi r^2$

Surface Area Formulas

Shapes	Surface Areas
Cuboid	$2(lb + bh + hl)$
Cube	$6a^2$
Right Circular Cylinder	$2\pi r(r + h)$
Right Circular Cone	$\pi r(l + r)$, ($l^2 = h^2 + r^2$)
Sphere	$4 \pi r^2$

Volume of a Cuboid

The volume of a cuboid is the product of its dimensions.

Volume of a cuboid = length × breadth × height= lbh

Where l is the length of the cuboid, b is the breadth, and h is the height of the cuboid.

Volume of a Cube

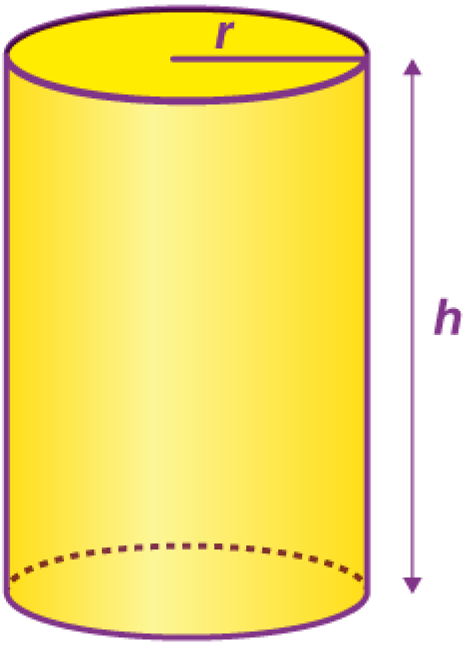
The volume of a cube = base area × height.

Since all dimensions are identical, the volume of the cube = a^3

Where a is the length of the edge of the cube.

Volume of a Right Circular Cylinder

The volume of a right circular cylinder is equal to base area × its height.



The volume of cylinder $=\pi r^2 h$

Where r is the radius of the base of the cylinder and h is the height of the cylinder.

Volume of a Right Circular Cone

The volume of a Right circular cone is 1/3 times the volume of a cylinder with the same radius and height. In other words, three cones make one cylinder of the same height and base.

The volume of right circular cone = $(1/3)\ \pi r^2 h$

Where r is the radius of the base of the cone and h is the height of the cone.

Volume of a Sphere

The volume of a sphere of radius r = $(4/3)\pi r^3$

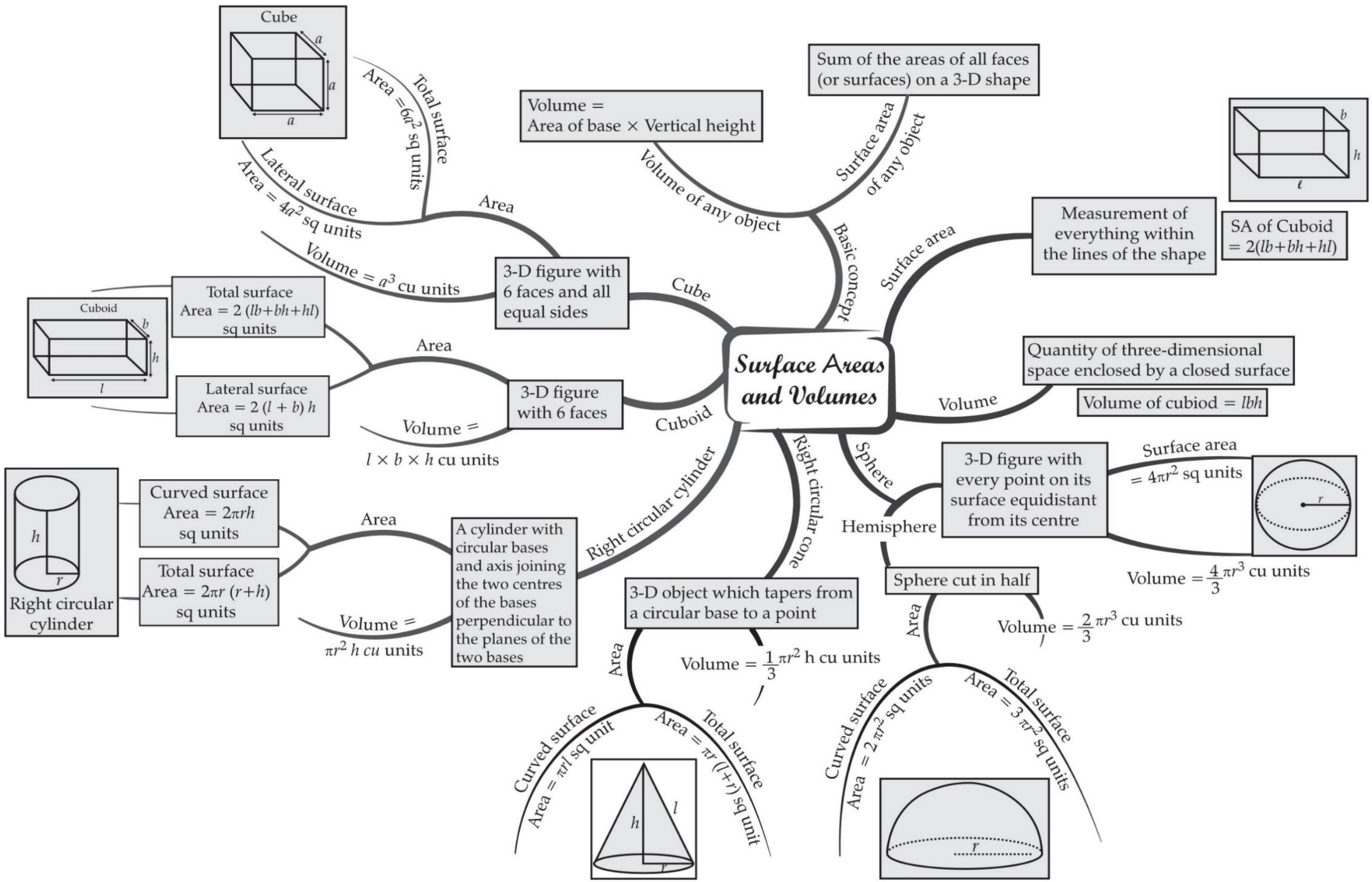
Volume Formulas

Shapes	Volumes
Cuboid	length × breadth × height
Cube	a^3
Right Circular Cylinder	$\pi r^2 h$
Right Circular Cone	⅓ $\pi r^2 h$
Sphere	$4/3\ \pi\ r^3$

Volume and Capacity

The volume of an object is the measure of the space it occupies, and the capacity of an object is the volume of substance its interior can accommodate. The unit of measurement of either volume or capacity is a cubic unit.

CHAPTER : 13 SURFACE AREAS AND VOLUMES



Important Questions

Multiple Choice questions-

Question 1. If the perimeter of one of the faces of a cube is 40cm, then its volume is:

(a) $6000cm^3$

(b) $1600cm^3$

(c) $1000cm^3$

(d) $600cm^3$

Question 2. A cuboid having surface areas of 3 adjacent faces as a, b and c has the volume:

(a) $3\sqrt{abc}$

(b) $\sqrt{abc}$

(c) ABC

(d) $(ABC)^2$

Question 3. The radius of a cylinder is doubled, and the height remains the same. The ratio between the volumes of the new cylinder and the original cylinder is

(a) 1 : 2

(b) 3 : 1

(c) 4 : 1

(d) 1 : 8

Question 4. Length of diagonals of a cube of side a cm is

(a) $\sqrt{2}a$ cm

(b) $\sqrt{3}a$ cm

(c) $\sqrt{3a}$ cm

(d) 1cm

Question 5. Volume of spherical shell is

(a) $\frac{2}{3}\pi r^3$

(b) $\frac{3}{4}\pi r^3$

(c) $\frac{4}{3}\pi(R^3 - r^3)$

(d) None of these

Question 6. Volume of hollow cylinder

(a) $\pi(R^2 - r^2)h$

(b) $\pi R^2 h$

(c) $\pi r^2 h$

(d) $\pi r^2(h_1 - h_1)$

Question 7. The radius of a sphere is 2r, then its volume will be

(a) $\frac{4}{3}\pi r^3$

(b) $4\pi r^3$

(c) $\frac{8}{3}\pi r^3$

(d) $\frac{32}{3}\pi r^3$

Question 8. In a cylinder, radius is doubled, and height is halved, curved surface area will be

(a) Halved

(b) Doubled

(c) Same

(d) Four time

Question 9. The total surface area of a cone whose radius is r^2 and slant height 2l is

(a) $2\pi r(l + r)$

(b) $\pi r(l + \frac{r}{4})$

(c) $\pi r(l + r)$

(d) $2\pi rl$

Question 10. The radius of a hemispherical balloon increases from 6cm to 12cm as air is being pumped into it. The ratios of the surface areas of the balloon in the two cases is

(a) 1 : 4

(b) 1 : 3

(c) 2 : 3

(d) 2 : 1

Very Short:

1. How much ice-cream can be put into a cone with base radius 3.5cm and

height 12cm?

2. Calculate the edge of the cube if its volume is $1331cm^3$.
3. The curved surface area of a cone is 12320 sq. cm, if the radius of its base is 56cm, find its

   height.
4. Two cubes of edge 6cm are joined to form a cuboid. Find the total surface area of the cuboid.
5. A metallic sphere is of radius 4.9cm. If the density of the metal is $7.8\ g/cm^2$, find the mass of the sphere $(\pi = \frac{22}{7})$
6. The volume of a solid hemisphere is $1152\ \pi\ cm^3$. Find its curved surface area.
7. Find the diameter of a cylinder whose height is 5cm and numerical value of volume is equal to numerical value of curved surface area.
8. In a cylinder, if radius is halved and height is doubled, then find the volume with respect to original volume.

Short Questions:

1. A spherical ball is divided into two equal halves. If the curved surface area of each half is 56.57cm, find the volume of the spherical ball. [use π = 3.14]
2. Find the capacity in liters of a conical vessel having height 8 cm and slant height 10cm.
3. Calculate the surface area of a hemispherical dome of a temple with radius 14m to be whitewashed from outside
4. A rectangular piece of paper is 22cm long and 10cm wide. A cylinder is formed by rolling the paper along its length. Find the volume of the cylinder.
5. A heap of wheat is in the form of a cone whose diameter is 10.5m and height is 3m. Find it volume. If 1m3 wheat cost is ₹ 10, then find total cost.
6. A cylindrical vessel can hold 154 g of water. If the radius of its base is 3.5cm, and $1cm^3$ of water weighs 1 g, find the depth of water.

Long Questions:

1. It costs ₹ 3300 to paint the inner curved surface of a 10m deep well. If the rate cost of

   painting is of ₹ 30 per m^2, find:

   (a) inner curved surface area

(b) diameter of the well

(c) capacity of the well.

2. Using clay, Anant made a right circular cone of height 48cm and base radius 12cm. Varsha reshapes it in the form of a sphere. Find the radius and curved surface area of the sphere so formed.
3. A dome of a building is in the form of a hemisphere. From inside, it was whitewashed at the cost of ₹ 498.96. If the rate of whitewashing is ₹ 4 per square metre, find the:

   (i) Inside surface area of the dome

   (ii) Volume of the air inside the dome.
4. A right triangle ABC with sides 5cm, 12cm and 13cm is revolved about the side 5cm. Find the volume of the solid so obtained. If it is now revolved about the side 12cm, then what would be the ratio of the volumes of the two solids obtained in two cases?
5. A right triangle of hypotenuse 13cm and one of its sides 12cm is made to revolve taking side 12cm as its axis. Find the volume and curved surface area of the solid so formed.

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: If diameter of a sphere is decreased by 25%, then its curved surface area is decreased by 43.75%.

Reason: Curved surface area is increased when diameter decreases.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: The external dimensions of a wooden box are 18 cm, 10 cm and 6 cm respectively and thickness of the wood is 15 mm, then the internal volume is 765 cm^3.

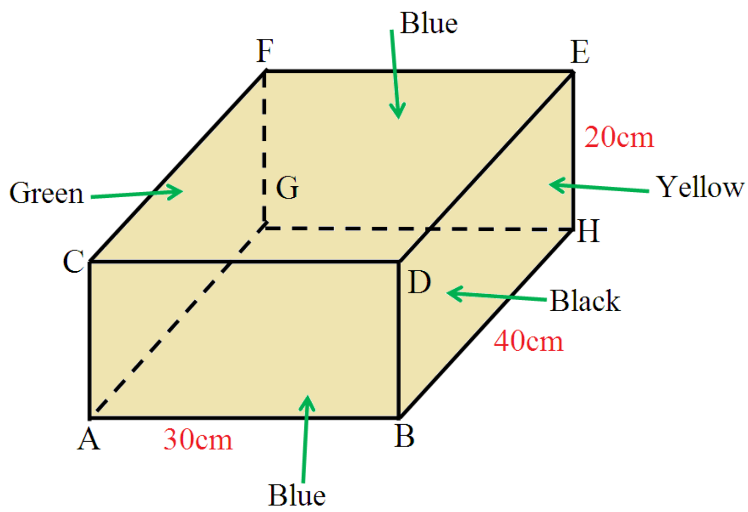
Reason: If external dimensions of a rectangular box be l, b and h and the thickness of its sides be x , then its internal volume is (l – 2x) (b – 2x) (h – 2x)

Case Study Questions-

1. Read the Source/ Text given below and answer any four questions:

Veena planned to make a jewellery box to gift her friend Reeta on her marriage. She made the jewellery box of wood in the shape of a cuboid. The jewellery box has the dimensions as shown in the figure below. The rate of painting the exterior of the box is Rs. 2 per cm^2. After making the box she took help from his friends to decorate the box.

The blue colour was painted by Deepak, Black by Suresh, green by Harsh and the yellow was painted by Naresh.



i. What is the volume of the box?

a. 24000cm^3

b. 1200cm^3

c. 800cm^3

d. 600cm^3

ii. How much area did Suresh paint?

a. 24000cm^2

b. 1200cm^2

c. 800cm^2

d. 600cm^2

iii. How much area did Deepak paint?

a. $24000cm^2$

b. $600cm^2$

c. $800cm^2$

d. $1200cm^2$

iv. What amount did Harsh charge?

a. Rs. 800

b. Rs. 1200

c. Rs. 1600

d. Rs. 2000

v. What amount did Veena pay for painting:

a. Rs. 2600

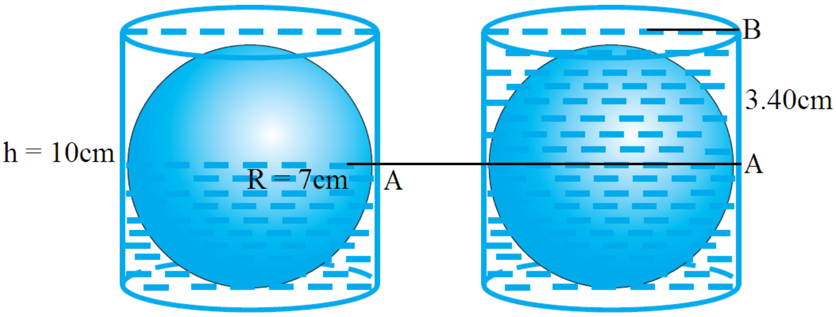
b. Rs. 5200

c. Rs. 5000

d. Rs. 6000

2. Read the passage given below and answer these questions:

Dev was doing an experiment to find the radius r of a sphere. For this he took a cylindrical container with radius R = 7cm and height 10cm. He filled the container almost half by water as shown in the left figure. Now he dropped the yellow sphere in the container. Now he observed as shown in the right figure the water level in the container raised from A to B equal to 3.40cm.



i. What is the approximate radius of the sphere?

a. 7cm

b. 5cm

c. 4cm

d. 3cm

ii. What is the volume of the cylinder?

a. $700cm^3$

b. $500cm^3$

c. $1540cm^3$

d. $2000cm^3$

iii. What is the volume of the sphere?

a. $700cm^3$

b. $600cm^3$

c. $500cm^3$

d. $523.8cm^3$

iv. How many litres water can be filled in the full container? (Take 1 litre = $1000cm^3$):

a. 1.50

b. 1.44

c. 1.54

d. 2

v. What is the surface area of the sphere?

a. $314.3m^2$

b. $300m^2$

c. $400m^2$

d. $350m^2$

Answer Key:

MCQ:

1. (c) $1000cm^3$
2. (b) $\sqrt{abc}$
3. (c) 4 : 1
4. (b) $\sqrt{3}a$ cm
5. (c) $\frac{4}{3}\pi(R^3 - r^3)$
6. (a) $\pi(R^2 - r^2)h$
7. (d) $\frac{32}{3}\pi r^3$
8. (c) Same
9. (b) $\pi r(l + \frac{r}{4})$
10. (a) 1 : 4

Very Short Answer:

1. Here, radius (r) = 3.5cm and height (h) = 12cm

   ∴ Amount of ice-cream = $\frac{1}{3}\pi r^2$

   $= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 12$

   = $154cm^3$

2. Volume of cube = $1331cm^3$

   $(Side)^3 = 1331$

   Side = $(11 \times 11 \times 11)^{\frac{1}{3}}$ = 11cm

3. Here, radius of base of a cone (r) = 56cm

   And, curved surface area = $12320cm^2$

   $\pi rl = 12320$

   $l = \frac{12320}{\pi r}$

   $= \frac{12320 \times 7}{22 \times 56} = 70\text{cm}$

   Again, we have

   $r^2 + h^2 = l^2$

   $h^2 = l^2 - r^2 = 70^2 - 56^2$

   = 4900 - 3136 = 1764

   h = $\sqrt{1764}$ = 42cm

   Hence, the height of the cone is 42cm.

4. When two cubes are joined end to end, then

   Length of the cuboid = 6 + 6 = 12cm

   Breadth of the cuboid = 6cm

   Height of the cuboid = 6cm

   Total surface area of the cuboid = 2 (lb + bh + hl)

   = 2(12 × 6 + 6 × 6 + 6 × 12)

   = 2(72 + 36 + 72) = 2(180)

   = $360cm^2$

5. Here, radius of metallic sphere (r) = 4.9cm

$$\therefore \quad \text{Volume of metallic sphere} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\times\frac{22}{7}\times(4.9)^3$$
$$= \frac{4}{3}\times\frac{22}{7}\times 4.9\times 4.9\times 4.9$$
$$= 493 \text{ cm}^3$$
$$\text{Density of metal used} = 7.8 \text{ g/cm}^3$$
$$\text{Mass of the sphere} = \text{Volume} \times \text{Density}$$
$$= 493 \times 7.8 = 3845.4 \text{ g}$$

6. Here, volume of hemisphere = 1152π cm^3

$\therefore \frac{2}{3}\pi r3 = 1152$

$\Rightarrow r^3 = (12)^3 \pi$

$\Rightarrow r \frac{1152 \times 3}{3} = 1728$

$\Rightarrow r^3 = (12)^3$

Now, curved surface area = $2\pi r^2$

$= 2 \times \pi \times (12)^2 = 288\pi$ cm^2

7. Here, height of cylinder (h) = 5cm

According to the statement of the question, we have

$\pi r^2 h = 2\pi r h$

r = 2cm

Thus, diameter of the base of the cylinder is 2 × 2 i.e., 4cm.

8. Here, $r = \frac{r}{2}$. h = 2h

$$\therefore \quad \text{Volume of cylinder} = \pi\left(\frac{r}{2}\right)^2 2h = \frac{1}{2}\pi r^2 h$$
$$\text{Original volume of cylinder} = \pi r^2 h$$

Volume w.r.t. original volume of cylinder

$$= \frac{\frac{1}{2}\pi r^2 h}{\pi r^2 h} = \frac{1}{2}$$

Short Answer:

Ans: 1. Since curved surface of half of the spherical ball = 56.57cm^2

$2\pi r^2 = 56.57$

$$\Rightarrow r^2 = \frac{56.57}{2 \times 3.14} = 9$$

$$\Rightarrow r = 3 \text{ cm}$$

Now, volume of spherical ball $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times 3.14 \times 3 \times 3 \times 3$$

$$= 113.04 \text{ cm}^3$$

$= 113.04\text{cm}^3$

Ans: 2. Height of conical vessel (h) = 8cm

Slant height of conical vessel (l) = 10cm

$\therefore r^2 + h^2 = l^2$

$\Rightarrow r^2 + 82 = 10^2$

$\Rightarrow r^2 = 100 - 64 = 36$

$\Rightarrow r = 6\text{cm}$

Now, volume of conical vessel $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 6 \times 8 = 301.71\text{cm}^3 = 0.30171$ litre

Ans: 3. Here, radius of hemispherical dome (r) = 14m

Surface area of dome $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 14 \times 14 = 1232\text{m}^2$

Hence, total surface area to be whitewashed from outside is 1232m^2.

Ans: 4. Since rectangular piece of paper is rolled along its length.

$\therefore 2\pi r = 22$

$r = \frac{22 \times 7}{2 \times 22} = 3.5\text{cm}$

Height of cylinder (h) = 10cm

$\therefore$ Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 3.5 \times 3.5 \times 10 = 385\text{cm}^3$

Ans: 5. Diameter of cone = 10.5m

Radius of cone (r) = 5.25m

Height of cone (h) = 3m

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 3$

$= 86.625 m^3$

Cost of $1m^3$ of wheat = ₹ 10

Cost of 86.625 m3 of wheat = ₹ 10 × 86.625

= ₹ 866.25

Ans: 6. Since 1 cm3 of water weighs 1 g.

∴ Volume of cylindrical vessel = 154 cm3

πr2h = 154

227 × 3.5 × 3.5 × h = 154

$h = \frac{154 \times 7}{22 \times 3.5 \times 3.5}$

h = 4

cm Hence, the depth of water is 4cm.

Long Answer:

Ans: 1. Depth of well (h) = 10m

Cost of painting inner curved surface is ₹ 30 per m^2 and total cost is ₹ 3300

$\therefore$ Curved surface area $= \frac{3300}{30} = 110 \text{ m}^2$

$$2\pi rh = 110$$

$$r = \frac{110}{2\pi h}$$

$$= \frac{110 \times 7}{2 \times 22 \times 10}$$

$$= 1.75 \text{ m} = 175 \text{ cm}$$

Now, volume of the well $= \pi r^2 h$

$$= \frac{22}{7} \times 1.75 \times 1.75 \times 10 = 96.25 \text{ m}^3$$

Hence, inner curved surface area is 110m^2, diameter of the well is 2 × 1.75 i.e., 3.5m and capacity of the well is 96.25m^3.

Ans: 2. Height of cone (h) = 48 cm

Radius of the base of cone = 12 cm

Let R be the radius of sphere so formed

∴ Volume of sphere = Volume of cone

$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$

$4R^3 = 12 \times 12 \times 48$

$R^3 = 12 \times 12 \times 12$

R = 12cm

Now, curved surface area of sphere $= 4\pi R^2$

$= 4 \times \frac{22}{7} \times 12 \times 12$

= 1810.29cm

Ans: 3. Here, dome of building is a hemisphere.

Total cost of whitewashing inside the dome = ₹ 498.96

Rate of whitewashing = ₹ 4 per m^2

$\therefore$ Inside surface area of the dome $= \frac{498.96}{4} = 124.74 \text{ m}^2$

$\therefore$ $2\pi r^2 = 124.74$

$$2 \times \frac{22}{7} \times r^2 = 124.74$$

$$r^2 = \frac{124.74 \times 7}{2 \times 22}$$

$$\Rightarrow \quad r^2 = 19.845$$

$$\Rightarrow \quad r = 4.45 \text{ cm}$$

$$\text{Volume of the air inside the dome} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 4.45 \times 4.45 \times 4.45$$

$$= 184.63 \text{ cm}^3$$

Ans: 4. Here, right triangle ABC with sides 5cm, 12cm and 13cm is revolved about the side 5cm.

∴ Radius of the base of cone = 12cm

Height of the cone = 5cm

$$\therefore \quad \text{Volume of the cone} = \frac{1}{3}\pi(12)^2(5) = \frac{\pi}{3} \times 720 \text{ cm}^3$$

Again, right triangle ABC is now revolved about the side 12 cm.

∴ Radius of the base of cone = 5 cm

Height of the cone = 12 cm

$$\therefore \quad \text{Volume of the cone} = \frac{1}{3}\pi(5)^2(12) = \frac{\pi}{3} \times 300 \text{ cm}^3$$

$$\text{Now, the required ratio of their volumes} = \frac{\pi}{3} \times 720 : \frac{\pi}{3} \times 300$$

= 12 : 5

Ans: 5. Here, hypotenuse and one side of a right triangle are 13cm and 12cm respectively

$$\therefore \quad \text{Third side} = \sqrt{(13)^2 - (12)^2}$$

$$= \sqrt{169 - 144}$$

$$= \sqrt{25} = 5 \text{ cm}$$

Now, given triangle is revolved, taking 12cm as its axis

∴ Radius of the cone (r) = 5cm

Height of the cone (h) = 12cm

Slant height of the cone (1) = 13cm

∴ Curved surface area = $\pi rl = \pi(5)(13) = 65\pi cm^2$

Volume of the cone = $\frac{1}{2}\pi r^2 h = \frac{1}{2}\pi \times 5 \times 5 \times 12 = 100\pi \text{ cm}^3$

Hence, the volume and curved surface area of the solid so formed are 100 π cm^3 and 65 π cm^2 respectively.

Assertion and Reason Answers-

1. c) Assertion is correct statement but reason is wrong statement.

2. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Case Study Answers-

1.

(i)	(a)	$24000cm^3$
(ii)	(b)	$1200cm^2$
(iii)	(d)	$1200cm^2$
(iv)	(c)	Rs. 1600
(v)	(b)	Rs. 5200

2.

(i)	(b)	5cm
(ii)	(c)	$1540cm^3$
(iii)	(d)	$523.8cm^3$
(iv)	(c)	1.54
(v)	(a)	$314.3m^2$