MATHEMATICS

Chapter 12: Exponents And Powers

Exponents And Powers

1. Large numbers can be written in shorter form using exponents.

   For example: 1000 = 10^3

   Here, 10^3 is called the exponential form of 1000.

   10 (the number that is being multiplied) is called the base.

   3 (number of times the same number is multiplied by itself) is called the power (or index or exponent)

2. As the exponent increases by 1 the value becomes ten times the previous value.

3. As the exponent decreases by 1 the value becomes $\frac{1}{10}$th the previous value.

4. For any non-zero integer 'a', $a^{-m} = \frac{1}{a^m}$, where m is a natural number.

5. For a and b non-zero rational numbers, then $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}$, where m is a natural number.

6. **Laws of exponents**: If 'a' and 'b' are rational numbers different from zero and if x, y are positive integers, then

   i. $a^x \times a^y = a^{x+y}$

   ii. $a^x \div a^y = a^{x-y}$

   iii. $(a^x)^y = a^{xy}$

   iv. $(ab)^x = a^x + b^x$

   v. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

   vi. $\left(\frac{a}{b}\right)^{-x} \left(\frac{b}{a}\right)^x$

   vii. $a^0 = 1$

   viii. (-1) odd number = -1
   (-1) even number = 1

7. **Exponential Equation**: An equation which has an unknown quantity as an exponent is called an exponential equation.

   Example: (i) $5^x = 625$ (ii) $3^{3x-15} = 1$

8. A number is said to be in standard form (or scientific notation) if it can be written as ($k \times 10^n$), where k is real number such that $1 \le k < 10$, and n is a positive integer.

   Example:

   i. 160000 = (1.6×10^5)

ii. 1548000 = (1.548×10^6)

iii. 0.0016 = (1.6×10^{-3})

9. To write very small numbers in standard form:
   - Get the number first and check if it lies between 1 and 10 or less than 1.
   - When the number is between 1 and 10, then write it as a product of the number itself and 10^0.
   - When the number is less than 1, then shift the decimal point to the right such that there is only one digit on the left side of the decimal point. Now write the given number as the product of the number so obtained and 10^{-n}, where n is the number of places the decimal point has been shifted to the right. Thus, the final number so obtained is the standard form of the given number.

Powers and Exponents

The power of a number indicates the number of times it must be multiplied. It is written in the form ab. Where 'b' indicates the number of times 'a' needs to be multiplied to get our result. Here 'a' is called the base and 'b' is called the exponent.

For example: Consider 9^3. Here the exponent '3' indicates that base '9' needs to be multiplied three times to get our equivalent answer which is 27.

General Form of Exponents

The exponent is a simple but powerful tool. It tells us how many times a number should be multiplied by itself to get the desired result. Thus any number 'a' raised to power 'n' can be expressed as:

$$a^n = \underbrace{a \times a \times a \times \ldots\ldots\ldots \times a}_{n-times}$$

Here a is any number and n is a natural number.

a^n is also called the nth power of a.

'a' is the base and 'n' is the exponent or index or power.

'a' is multiplied 'n' times, and thereby exponentiation is the shorthand method of repeated multiplication.

Exponents and Powers Applications

Scientific notation uses the power of ten expressed as exponents, so we need a little background before we can jump in. In this concept, we round out your knowledge of exponents, which we studied in previous classes.

The distance between the Sun and the Earth is 149,600,000 kilometres. The mass of the Sun is 1,989,000,000,000,000,000,000,000,000,000 kilograms. The age of the Earth is 4,550,000,000 years. These numbers are way too large or small to memorize in this way.

With the help of exponents and powers, these huge numbers can be reduced to a very compact form and can be easily expressed in powers of 10.

Now, coming back to the examples we mentioned above, we can express the distance between the Sun and the Earth with the help of exponents and powers as following:

Distance between the Sun and the Earth 149,600,000 = 1.496 × 10 × 10 × 10 × 10 × 10× 10 × 10 = 1.496 × 10^8 kilometers.

Mass of the Sun: 1,989,000,000,000,000,000,000,000,000,000 kilograms = 1.989 × 10^{30} kilograms.

Age of the Earth: 4,550,000,000 years = 4. 55 × 10^9 years

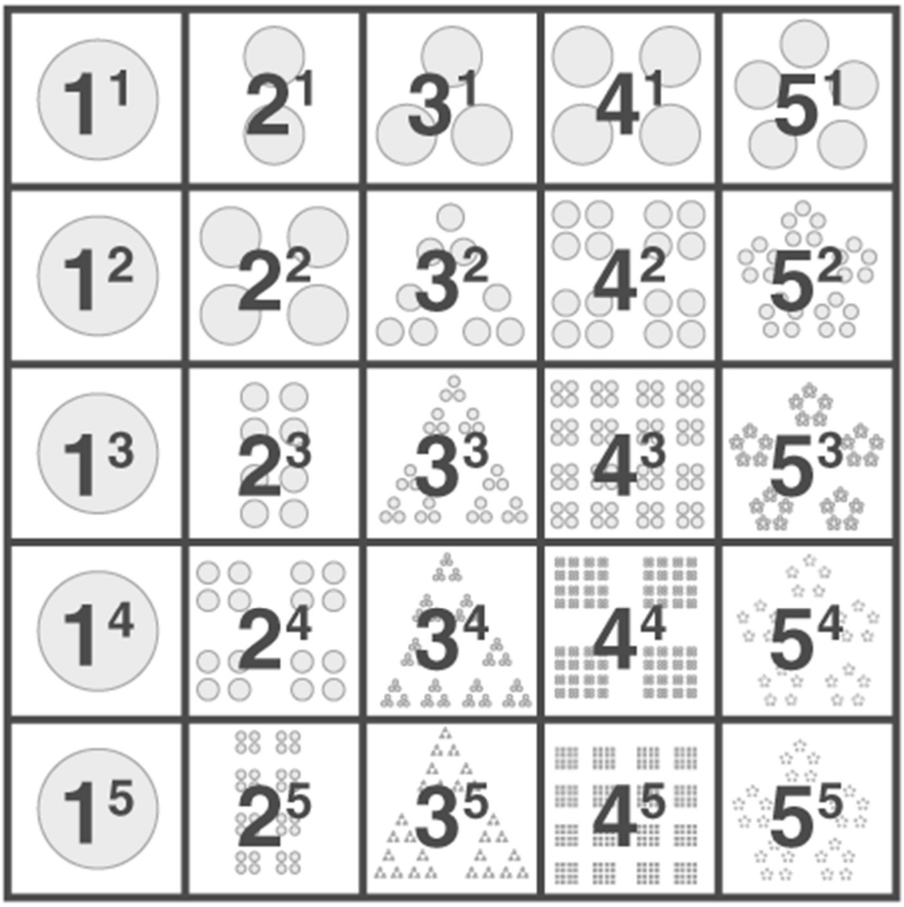
Powers with Negative Exponents

A negative exponent in power for any non-integer is basically a reciprocal of the power.

In simple terms, for a non-zero integer a with an exponent -b, $a^{-b} = 1^{ab}$

Visualising Powers and Exponents

Powers of numbers can easily be visualized in the form of shapes and figures. Consider the following visulization.



Expanding a Rational Number Using Powers

A given rational number can be expressed in expanded form with the help of exponents.

Consider a number 1204.65. When expanded the number can be written like, 1204.65 = 1000 + 200 + 4 + 0.6 + 0.05 = $(1 \times 10^3) + (2 \times 10^2) + (0 \times 10^1) + (4 \times 10^{-1}) + (5 \times 10^{-2})$

Laws of Exponents

Exponents with like Bases

Given a non-zero integer a, $a^m \times a^n = a^{m+n}$ where m and n are integers.

and $a^m \div a^n = a^{m-n}$ where m and n are integers.

For example: $2^3 \times 2^7 = 2^{7+3} = 2^{10}$

and $2^7 2^3 = 2^{7-3}$

Power of a Power

Given a non-zero integer a, $(a^m)^n = a^{mn}$, where m and n are integers.

For example: $(24)^3 = 2^{4\times3} = 2^{12}$ Given a non-zero integer a,

$(a)^0 = 1$ Any number to the power 0 is always 1.

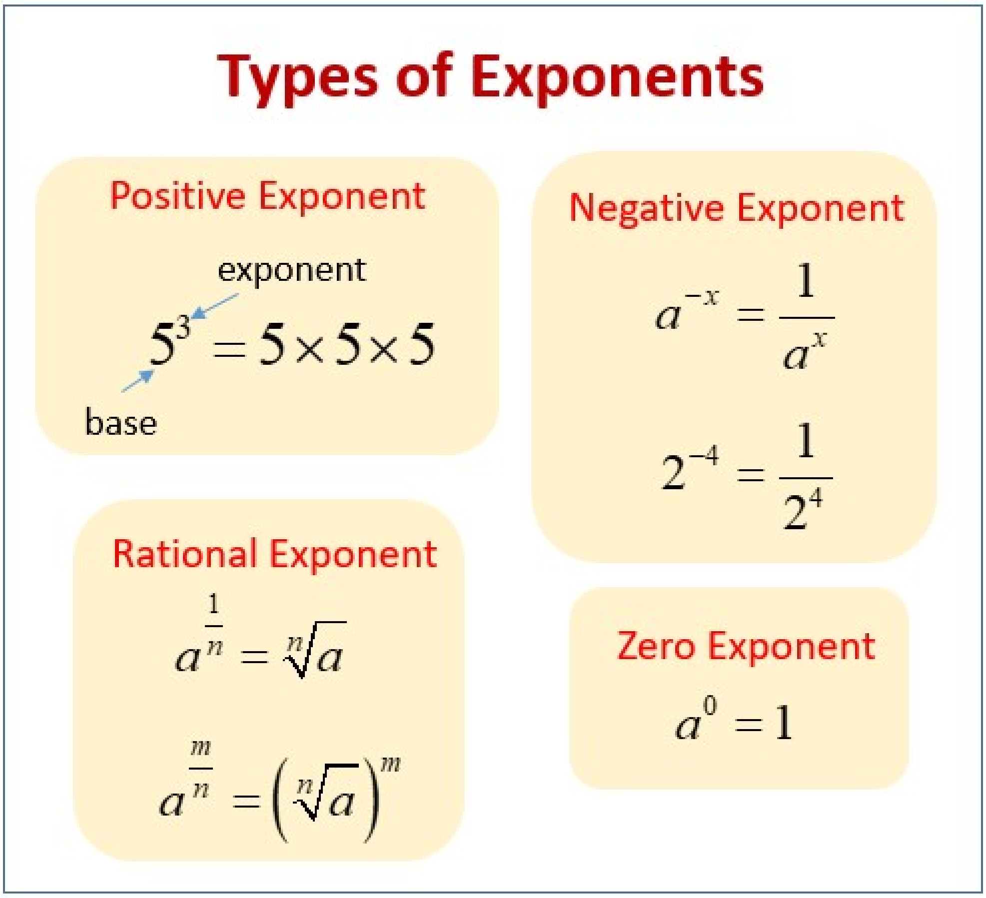
Exponents with Unlike Bases and Same Exponent

Given two non-zero integers a and b,

$a^m \times b^m = (a \times b)^m$, where m is an integer.

For example: $2^3 \times 5^3 = (2 \times 5)^3 = 10^3 = 1000$

Types of Exponents



Exponents can be divided into four types based on the number in the power. They are:

Positive exponent

Negative exponent

Zero exponent

Rational exponent

Positive Exponents

Positive exponents can be simplified just by multiplying the base to itself the number of times indicated by the exponent/power.

Negative Exponents

A negative exponent can be simplified by placing 1 in the numerator and the base along with the exponent in the denominator of a fraction.

Zero Exponents

Zero exponents Any expression with the exponent as 0 is equal to 1 and no need to consider the base value during simplification.

Rational Exponents

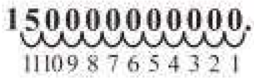
Rational or fractional exponents will become radical or roots. For example, 31/3 can be written as 3root of 3, 65/2 can be written as 2 root (or square root) of 6 raises to the power 5.

Uses of Exponents

Inter Conversion between Standard and Normal Forms

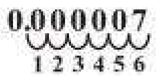
Very large numbers or very small numbers can be represented in the standard form with the help of exponents.

If it is a very large number like 150,000,000,000, then we need to move the decimal place towards the left. And when we do so the exponent will be positive.



Since the decimal is moved 11 places till it is placed between 1 and 5, our standard form representation of the large number will be 1.5×1011

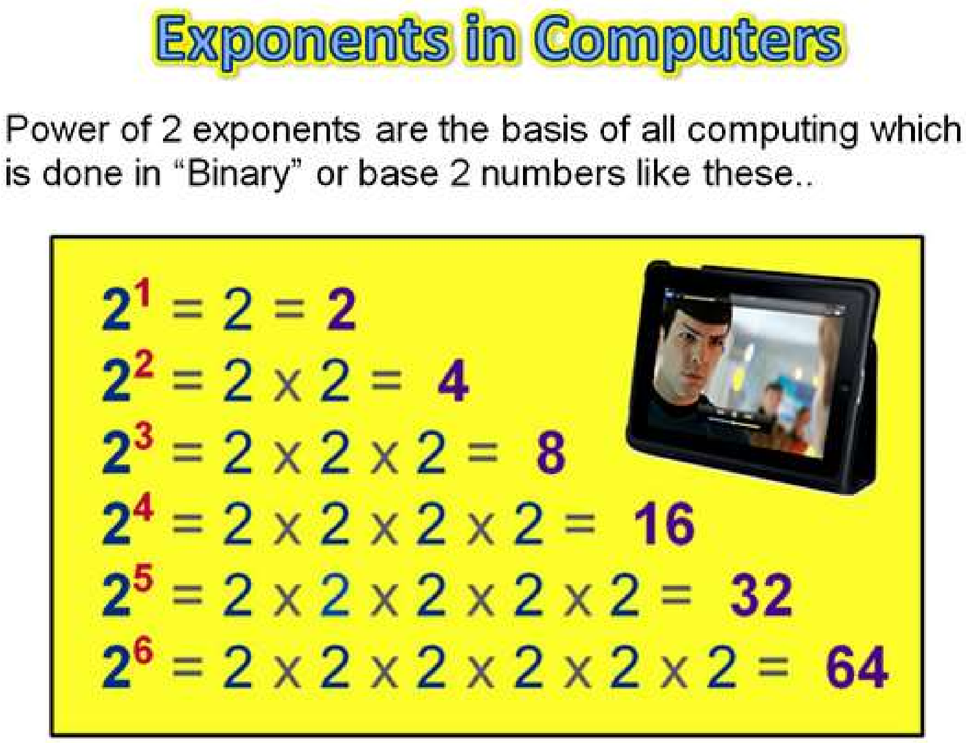
If it is a very small number like 0.000007, we need to move the decimal places to the right in-order to represent the number in its standard form. When being shifted to the right, the exponent will be negative.



In this case, the decimal place is moved 6 places up until till it is placed after digit 7. Therefore, our standard form representation will be

7×10^{-6}

The exponents are also useful when converting the number from it’s standard form to it’s natural form.



Exponents and Viral Marketing

If One Person , tells another 10 people, and then each of these 10 people tell another 10 people, and so on, we get rapid spreading of a message, video, photo, news item, or product across the Internet.

Level	0		1		2		3		4	etc
Spread	1	+	10	+	100	+	1000	+	10 000	
Powers	10^0		10^1		10^2		10^3		10^4	

$$\text{Spread} = 10^{\text{Level}}$$

Exponents and pH Scale

The pH scale also uses Power of 10 Exponents, for how Acidic or Alkaline a substance is. Public Swimming Pools maintain a pH between 7.0 and 7.4 to provide the best possible comfort levels for the public, as well to ensure effective chlorine action.

Comparison of Quantities Using Exponents

In-order to compare two large or small quantities, we convert them to their standard exponential form and divide them.

For example: To compare the diameter of the earth and that of the sun.

Diameter of the Earth = 1.2756×10^6m

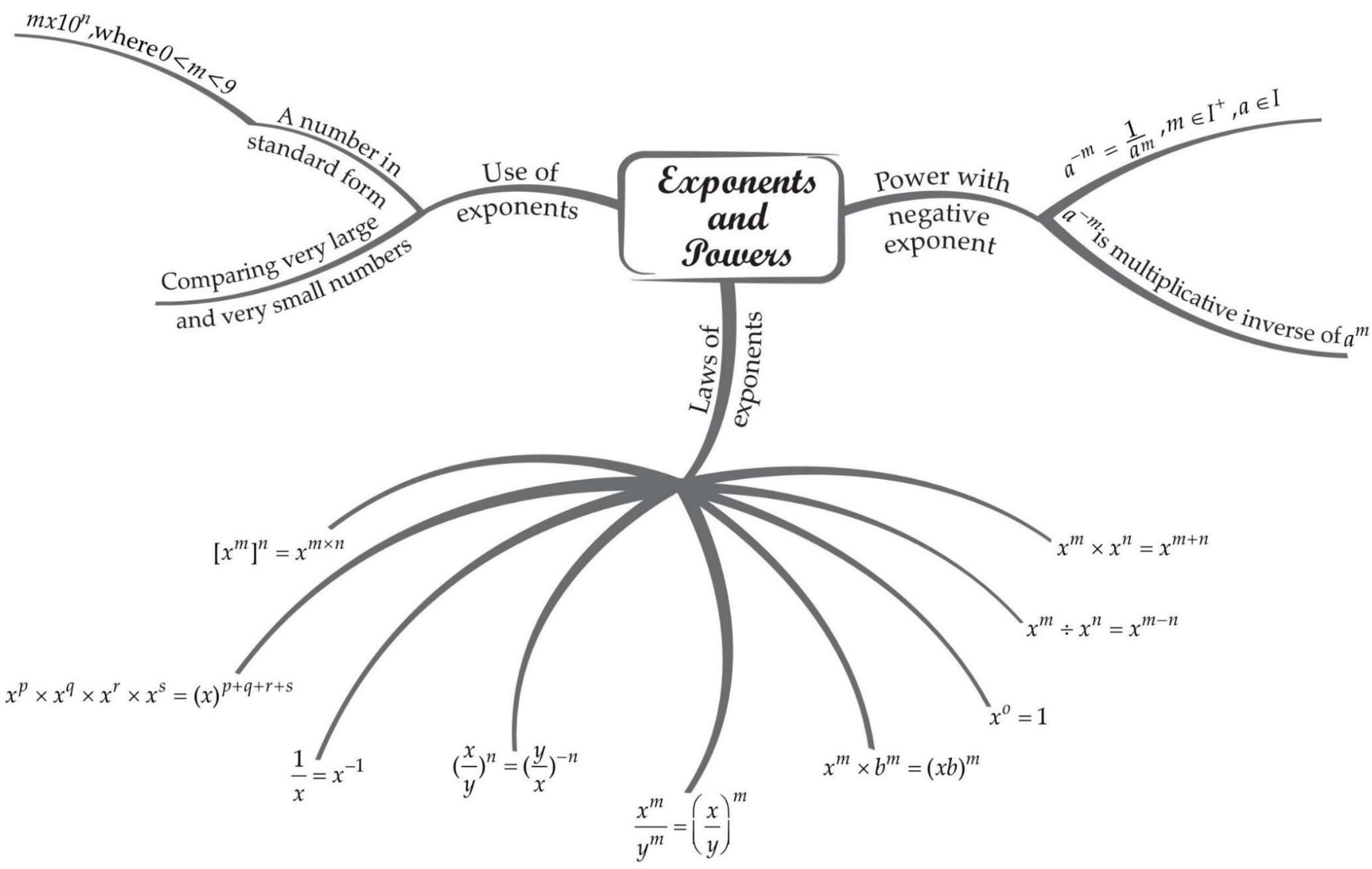
Diameter of the Sun $=1.4 \times 10^{9}$m

Diameter of the Earth $= 1.4 \times 10^{9}$m

1.2756×10^{7}m $= 109$

So the diameter of the Sun is 109 times that of the Earth! While calculating the total or the difference between two quantities, we must first ensure that the exponents of both the quantities are the same.

MIND MAP : LEARNING MADE SIMPLE CHAPTER-12



Important Questions

Multiple Choice Questions-

Question 1. $a^m \times a^m$ is equal to

(a) a^{m+n}

(b) a^{m-n}

(c) a^{mn}

(d) a^{n-m}

Question 2. $a^m \div a^n$ is equal to

(a) a^{m-n}

(b) a^{m+n}

(c) a^{mn}

(d) a^{n-m}

Question 3. $(a^m)^n$ is equal to

(a) a^{m+n}

(b) a^{m-n}

(c) a^{mn}

(d) a^{n-m}

Question 4. $a^m \times b^m$ is equal to

(a) $(ab)^m$

(b) $(ab)^{-m}$

(c) $a^m b$

(d) ab^m

Question 5. a^0 is equal to

(a) 0

(b) 1

(c) -1

(d) a.

Question 6. $\frac{a^m}{b^m}$ is equal to bm

(a) $\left(\frac{a}{b}\right)^m$

(b) $\left(\frac{b}{a}\right)^m$

(c) $\left(\frac{a^m}{b}\right)^m$

(d) $\left(\frac{a}{b^m}\right)^m$

Question 7. 2 × 2 × 2 × 2 × 2 is equal to

(a) 2^4

(b) 2^3

(c) 2^2

(d) 2^5

Question 8. In 10^2, the exponent is

(a) 1

(b) 2

(c) 10

(d) 1.

Question 9. In 10^2 the base is

(a) 1

(b) 0

(c) 10

(d) 100.

Question 10. 10^{-1} is equal to

(a) 10

(b) -1

(c) $\frac{1}{10}$

(d) $\frac{-1}{10}$

Very Short Questions:

1. Find the multiplicative inverse of:

(i) 3^{-3}

(ii) 10^{-10}

2. Expand the following using exponents.

(i) 0.0523

(ii) 32.005

3. Simplify and write in exponential form.

$(i)\ (-5)^2 \times (-5)^{-3}$ $(ii)\ \left(\frac{1}{2}\right)^{-3} \times \left(\frac{1}{2}\right)^{-2}$

4. Simplify the following and write in exponential form.

$(i)\ 2^3 \times 3^3$ $(ii)\ \left(\frac{4}{5}\right)^5 \times \left(\frac{5}{6}\right)^5$

5. Express 8-4 as a power with the base 2.

We have $8 = 2 \times 2 \times 2 = 2^3$

$8^{-4} = (2^3)^{-4} = 2^{3\times(-4)} = 2^{-12}$

Short Questions :

1. Simplify the following and write in exponential form.

(i) $(3^6 \div 3^8)^4 \times 3^{-4}$

(ii) $\frac{1}{27} \times 3^{-3}$

2. Find the value of k if $(-2)^{k+1} \times (-2)^3 = (-2)^7$

3. Simplify the following:

$(i)\ \left\{\left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

$(ii)\ \left(\frac{2}{3}\right)^{-6} \times \left(\frac{3}{2}\right)^{-4}$

4. Find the value of $\left[\left(-\frac{3}{4}\right)^{-2}\right]^2$

5. Write the following in standard form

(i) 0 0035

(ii) 365.05

6. Find the value of P if

$$\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^{-6} = \left(\frac{2}{5}\right)^{2P-1}$$

Long Questions :

1.

If $\left(\frac{x}{y}\right) = \left(\frac{3}{2}\right)^{-2} \div \left(\frac{3}{7}\right)^0$, find the value of $\left(\frac{x}{y}\right)^{-3}$.

2. Find the value of x if

$$\left(\frac{125}{27}\right)\times\left(\frac{125}{27}\right)^x=\left(\frac{5}{3}\right)^{18}$$

3. Solve the following: $(81)^{-4} \div (729)^{2-x} = 9^{4x}$

4.

Simplify: $$\frac{(x^{m+n})^2 \times (x^{n+p})^2 \times (x^{p+m})^2}{(x^m \cdot x^n \cdot x^p)^3}$$

5.

Simplify: $$\frac{(-2)^3 \times (-2)^7}{3 \times 4^6}$$

6. Find x so that $(-5)^{x+1} \times (-5)^5 = (-5)^7$

Answer Key-

Multiple Choice questions-

1. (a) a^{m+n}
2. (a) a^{m-n}
3. (c) a^{mn}
4. (a) $(ab)^m$
5. (b) 1
6. (a) $\left(\frac{a}{b}\right)^m$
7. (d) 2^5
8. (b) 2
9. (c) 10
10. (c) $\frac{1}{10}$

Very Short Answer :

1.

(i) Multiplicative inverse of $3^{-3} = \frac{1}{3^{-3}} = 3^3$

(ii) Multiplicative inverse of $10^{-10} = \frac{1}{10^{-10}} = 10^{10}$

2.

$$(i)\ 0.0523 = \frac{5}{100} + \frac{2}{1000} + \frac{3}{10000}$$

$$= 5 \times \frac{1}{100} + 2 \times \frac{1}{1000} + 3 \times \frac{1}{10000}$$

$$= 5 \times \frac{1}{10^2} + 2 \times \frac{1}{10^3} + 3 \times \frac{1}{10^4}$$

$$= 5 \times 10^{-2} + 2 \times 10^{-3} + 3 \times 10^{-4}$$

$$(ii)\ 32.005 = 3 \times 10 + 2 \times 1 + \frac{5}{1000}$$

$$= 3 \times 10 + 2 \times 1 + 5 \times \frac{1}{1000}$$

$$= 3 \times 10 + 2 \times 1 + 5 \times \frac{1}{10^3}$$

$$= 3 \times 10 + 2 \times 1 + \times 5 \times 10^{-3}$$

3.

$$(i)\ (-5)^2 \times (-5)^{-3}$$

$$= (-5)^{2+(-3)} = (-5)^{2-3}$$

$$= (-5)^{-1} = -\frac{1}{5}$$

$$(ii)\ \left(\frac{1}{2}\right)^{-3} \times \left(\frac{1}{2}\right)^{-2} = \left(\frac{1}{2}\right)^{(-3)+(-2)}$$

$$= \left(\frac{1}{2}\right)^{-3-2} = \left(\frac{1}{2}\right)^{-5}$$

$$= \frac{1}{2^{-5}} = 2^5$$

4.

$$(i)\ 2^3 \times 3^3 = (2 \times 3)^3 = 6^3$$

$$(ii)\ \left(\frac{4}{5}\right)^5 \times \left(\frac{5}{6}\right)^5 = \left(\frac{4}{5} \times \frac{5}{6}\right)^5$$

$$= \left(\frac{4}{6}\right)^5 = \left(\frac{2}{3}\right)^5$$

5. We have $8 = 2 \times 2 \times 2 = 2^3$

$8^{-4} = (2^3)^{-4} = 2^{3\times(-4)} = 2^{-12}$

Short Answer :

1.

(i) $(3^6 \div 3^8)^4 \times 3^{-4}$

$= (3^{6-8})^4 \times 3^{-4} = 3^{-2\times4} \times 3^{-4}$

$= 3^{-8} \times 3^{-4} = 3^{-8-4} = 3^{-12}$

(ii) $\frac{1}{27} \times 3^{-3}$

$= \frac{1}{3^3} \times 3^{-3}$

$= 3^{-3} \times 3^{-3} = 3^{-3-3} = 3^{-6}$

2. $(-2)^{k+1} \times (-2)^3 = (-2)^7$

$\Rightarrow (-2)^{k+1+3} = (-2)^7$

$\Rightarrow (-2)^{k+4} = (-2)^7$

$\Rightarrow k + 4 = 7$

$\Rightarrow k = 3$

Hence, k = 3.

3.

(i) $\left\{\left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

$= \left\{\frac{1^{-3}}{4^{-3}} - \frac{1^{-3}}{3^{-3}}\right\} \div \frac{1^{-2}}{4^{-2}}$

$\Rightarrow \left(\frac{4^3}{1^3} - \frac{3^3}{1^3}\right) \div \frac{4^2}{1^2}$

$\Rightarrow (64 - 27) \div 16$

$\Rightarrow 37 \div 16 = \frac{37}{16}$

(ii) $\left(\frac{2}{3}\right)^{-6} \times \left(\frac{3}{2}\right)^{-4}$

$= \frac{2^{-6}}{3^{-6}} \times \frac{3^{-4}}{2^{-4}} = \frac{3^6}{2^6} \times \frac{2^4}{3^4}$

$= \frac{3^{6-4}}{2^{6-4}} = \frac{3^2}{2^2} = \left(\frac{3}{2}\right)^2$

4.

$$\left[\left(-\frac{3}{4}\right)^{-2}\right]^2 = \left(-\frac{3}{4}\right)^{-4}$$

$$= (-1)^{-4} \times \left(\frac{3}{4}\right)^{-4}$$

$$= 1 \times \frac{3^{-4}}{4^{-4}} = \frac{4^4}{3^4} = \frac{256}{81}$$

5.

$$(i)\ 0.0035 = \frac{35}{10000}$$

$$= \frac{3.5 \times 10}{10^4} = 3.5 \times 10^{1-4} = 3.5 \times 10^{-3}$$

$$(ii)\ 365.05 = \frac{36505}{100} = \frac{3.6505 \times 10^4}{10^2}$$

$$= 3.6505 \times 10^2$$

6.

$$\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^{-6} = \left(\frac{2}{5}\right)^{2P-1}$$

$$\Rightarrow \left(\frac{2}{5}\right)^{3-6} = \left(\frac{2}{5}\right)^{2P-1}$$

$$\Rightarrow \left(\frac{2}{5}\right)^{-3} = \left(\frac{2}{5}\right)^{2P-1}$$

Equating the powers of the same base

$$2P - 1 = -3$$

$$2P = -3 + 1$$

$$2P = -2$$

$$\therefore \quad P = -1$$

Long Answer :

1.

$$\left(\frac{x}{y}\right) = \left(\frac{3^{-2}}{2^{-2}}\right) \div 1 \qquad \left[\because \left(\frac{a}{b}\right)^0 = 1\right]$$

$$= \frac{2^2}{3^2} = \frac{4}{9}$$

$$\therefore \quad \left(\frac{x}{y}\right)^{-3} = \left(\frac{4}{9}\right)^{-3}$$

$$= \frac{4^{-3}}{9^{-3}} = \frac{9^3}{4^3}$$

$$= \frac{729}{64}$$

2.

$$\left(\frac{125}{27}\right) \times \left(\frac{125}{27}\right)^x = \left(\frac{5}{3}\right)^{18}$$

$$\Rightarrow \quad \left(\frac{5^3}{3^3}\right) \times \left(\frac{5^3}{3^3}\right)^x = \left(\frac{5}{3}\right)^{18}$$

$$\Rightarrow \quad \left(\frac{5}{3}\right)^3 \times \left(\frac{5}{3}\right)^x = \left(\frac{5}{3}\right)^{18}$$

$$\Rightarrow \quad \left(\frac{5}{3}\right)^{3+x} = \left(\frac{5}{3}\right)^{18}$$

$\Rightarrow$ 3 + x = 18 [Equating the powers of same base]

x = 18 – 3 = 15

3.

$$(81)^{-4} \div (729)^{2-x} = (9)^{4x}$$

$$\Rightarrow \quad (9^2)^{-4} \div (9^3)^{2-x} = (9)^{4x}$$

$$\Rightarrow \quad 9^{-8} \div 9^{6-3x} = 9^{4x}$$

$$\Rightarrow \quad 9^{-8-(6-3x)} = 9^{4x}$$

$$\Rightarrow \quad 9^{-8-6+3x} = 9^{4x}$$

$$\Rightarrow \quad 9^{-14+3x} = 9^{4x}$$

Equating the power of same base, we have

$$-14 + 3x = 4x$$

$$\Rightarrow \quad 4x - 3x = -14$$

$$\therefore \quad x = -14$$

4.

$$\frac{(x^{m+n})^2 \times (x^{n+p})^2 \times (x^{p+m})^2}{(x^m \cdot x^n \cdot x^p)^3}$$

$$= \frac{x^{2m+2n} \times x^{2n+2p} \times x^{2p+2m}}{x^{3m} \cdot x^{3n} \cdot x^{3p}}$$

$$[\because \ (x^a)^b = (x^{ab})]$$

$$= \frac{x^{2m+2n+2n+2p+2p+2m}}{x^{3m+3n+3p}}$$

$$[\because \ x^a \times x^b = x^{a+b}]$$

$$= \frac{x^{4m+4n+4p}}{x^{3m+3n+3p}}$$

$$= x^{(4m+4n+4p)-(3m+3n+3p)}$$

$$= x^{4m+4n+4p-3m-3n-3p}$$

$$= x^{m+n+p} \qquad \left[\because \ \frac{x^a}{x^b} = x^{a-b}\right]$$

5.

$$\frac{(-2)^3 \times (-2)^7}{3 \times 4^6} = \frac{(-2)^{3+7}}{3 \times (2^2)^6} \quad \{a^m \times a^n = a^{m+n}\}$$

$$= \frac{(-2)^{10}}{3 \times 2^{12}} \quad \{(a^m)^n = a^{m \times n}\}$$

$$= \frac{(-2)^{10}}{3 \times 2^{12}} = \frac{2^{10-12}}{3} \quad \{a^m \div a^n = a^{m-n}, (-2)^{10} = 2^{10}\}$$

$$= \frac{2^{-2}}{3} = \frac{1}{3 \times 2^2} = \frac{1}{12}$$

6. $(-5)^{x+1} \times (-5)^5 = (-5)^7$

$(-5)^{x+1+5} = (-5)^7$ $\{a^m \times a^n = a^{m+n}\}$

$(-5)^{x+6} = (-5)^7$

On both sides, powers have the same base, so their exponents must be equal.

Therefore, x + 6 = 7

x = 7 – 6 = 1

x = 1.