MATHEMATICS

Chapter 9: Algebraic Expressions and Identities

Algebraic Expressions and Identities

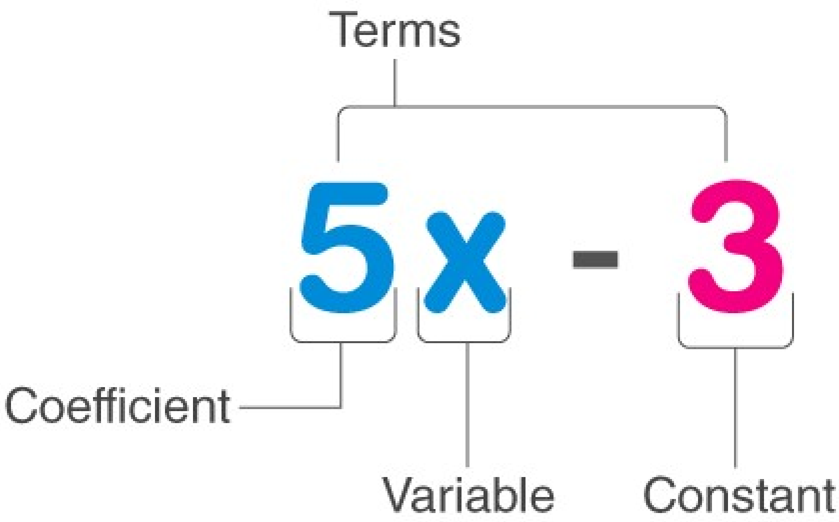
Algebraic Expressions

Algebraic expressions are expressions made up of variables and constants along with mathematical operators. Algebraic expressions have no sides or equal to sign like algebraic equations.

Examples of algebraic expressions are: $2x + 4$, $7y - 3 + 6x$, $3t^2 + 4t - 1$.

Variables, Coefficient & Constant in Algebraic Expressions

In Algebra we work with Variable, Symbols or Letters whose value is unknown to us.



In the above expression (i.e. $5x - 3$),

- x is a variable, whose value is unknown to us which can take any value.
- 5 is known as the coefficient of x, as it is a constant value used with the variable term and is well defined.
- 3 is the constant value term that has a definite value.

Factors

An algebraic expression is further divided into a product of one or more numbers and/or literals.

These numbers and literals are known as factors of that particular term.

Consider the example: $\frac{1}{3}xy^3$

Here, $\frac{1}{3}$, x and y are the factors of the term $\frac{1}{3}xy^3$

Numerical factor and Literal factor

Constant factor of the expression is called **numerical factor**, whereas the variable is called the literal factor.

Example: The expression, $2x - 6xy^3 + 12x^2z^2$, can be rewritten as $2x(1 - 3xy^3 + 6xz^2)$ Here, 2 is called the numerical factor and 'x' is called the literal factor.

Polynomial

An algebraic expression having one or more terms in which the variables involved have only non- negative integral powers is called a polynomial.

Example: $2 - 3x + 5x^2y - \frac{1}{3}xy^3$ is a polynomial.

Degree of a Polynomial

In case of a polynomial in one variable, the highest power of the variable is called the degree of the polynomial.

Eg: $5x^3 - 7x + \frac{3}{2}$ is a polynomial in x of degree 3.

In case of a polynomial having more than one variable, the sum of the power of the variables in each term is taken up and the highest sum so obtained is called the degree of the polynomial. Eg: $5x^3 - 2x^2y^2 - 3x^2y + 9y$ is a polynomial of degree 4 in x and y.

Other Types of Expression

Apart from monomial, binomial and polynomial types of expressions, an algebraic expression can also be classified into two additional types which are:

Numeric Expression

Variable Expression

Numeric Expression

A numeric expression consists of numbers and operations, but never include any variable. Some of the examples of numeric expressions are 10 + 5, 15 ÷ 2, etc.

Variable Expression

A variable expression is an expression that contains variables along with numbers and operation to define an expression. A few examples of a variable expression include 4x + y, 5ab + 33, etc.

Terms

Terms are the individual building blocks of expressions. They add up to form expressions. A term is a product of its factors.

For example, the expression 5xy – 3, is made up of two terms, 5xy and (-3).

Factors

Factors are those variables or constants, whose product form a term of an expression.

For example, 8, p and q are the factors of the term 8pq.

Factors are such that they cannot be factorized further.

The product of factors forms a term and the summation of the terms forms an expression.

Coefficients

The numerical factor of a term is called the coefficient of that term.

For the terms, 6y and 2xy, the coefficient of 6y is 6 and the coefficient of 2xy is 2.

Like Terms

Like terms are those terms which have same variables raised to the same power. Like terms have same algebraic factors. The numerical coefficient of like terms can be different.

For example, $3x^2y$ and $5x^2y$ are like terms.

Monomial

An expression with only one term is called a monomial.

Examples of monomials: 6x, 7pq, x^2y, 9xyz, 4bc etc.

Binomial

An expression which contains two unlike terms is called a binomial.

Examples of binomials: $4y - 3z$, $x^6 - 2$, $pq + 1$, etc.

Polynomial

Expressions that have more than two terms with non-zero coefficients and variables having non-negative integral exponents are called polynomials.

Examples: $a + b + c + 2$, $7xy - 8x + 2 + 3y$, $5t^3 - 7t + k + 3$.

Algebraic Identities

The algebraic equations which are valid for all values of variables in them are called algebraic identities. They are also used for the factorization of polynomials. In this way, algebraic identities are used in the computation of algebraic expressions and solving different polynomials. You have already learned about a few of them in the junior grades. In this article, we will recall them and introduce you to some more standard algebraic identities, along with examples.

Identity I

$(a + b)^2 = a^2 + 2ab + b^2$

Identity II

$(a - b)^2 = a^2 - 2ab + b^2$

Identity III

$a^2 - b^2 = (a + b)(a - b)$

Identity IV

$(x + a)(x + b) = x^2 + (a + b)x + ab$

Standard Algebraic Identities List

All the standard Algebraic Identities are derived from the Binomial Theorem, which is given as:

$$(a + b)^n = {}^nC_0 . a^n . b^0 + {}^nC_1 . a^{n-1} . b^1 + \ldots\ldots\ldots + {}^nC_{n-1} . a^1 . b^{n-1} + {}^nC_n . a^0 . b^n$$

Some Standard Algebraic Identities list are given below:

Identity I: $(a + b)^2 = a^2 + 2ab + b^2$

Identity II: $(a - b)^2 = a^2 - 2ab + b^2$

Identity III: a2 – b^2= (a + b) (a – b)

Identity IV: $(x + a) (x + b) = x^2 + (a + b) x + ab$

Identity V: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Identity VI: $(a + b)^3 = a^3 + b^3 + 3ab (a + b)$

Identity VII: $(a - b)^3 = a^3 - b^3 - 3ab (a - b)$

Identity VIII: $a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$

Example 1: Find the product of (x + 1) (x + 1) using standard algebraic identities.

Solution: (x + 1) (x + 1) can be written as $(x + 1)^2$. Thus, it is of the form Identity I where a = x and b = 1. So we have,

$(x + 1)^2 = (x)^2 + 2(x)(1) + (1)^2 = x^2 + 2x + 1$

Example 2: Factorise $(x^4 - 1)$ using standard algebraic identities.

Solution: $(x^4 - 1)$ is of the form Identity III where $a = x^2$ and b = 1. So we have,

$(x^4 - 1) = ((x^2)^2 - 1^2) = (x^2 + 1) (x^2 - 1)$

The factor $(x^2 - 1)$ can be further factorised using the same Identity III where a = x and b = 1. So,

$(x^4 - 1) = (x^2 + 1) ((x)^2 - (1)^2) = (x^2 + 1) (x + 1) (x - 1)$

Eample 3: Factorise $16x^2 + 4y^2 + 9z^2 – 16xy + 12yz – 24zx$ using standard algebraic identities.

Solution: $16x^2 + 4y^2 + 9z^2 – 16xy + 12yz – 24zx$ is of the form Identity V. So we have,

$16x^2 + 4y^2 + 9z^2 – 16xy + 12yz – 24zx = (4x)^2 + (-2y)^2 + (-3z)^2 + 2(4x) (-2y) + 2(-2y) (-3z) + 2(-3z) (4x) = (4x – 2y – 3z)^2 = (4x – 2y – 3z) (4x – 2y – 3z)$

Example 4: Expand $(3x – 4y)^3$ using standard algebraic identities.

Solution: $(3x– 4y)^3$ is of the form Identity VII where a = 3x and b = 4y. So we have,

$(3x – 4y)^3 = (3x)^3 – (4y)^3 – 3(3x) (4y) (3x – 4y) = 27x^3 – 64y^3 – 108x^2y + 144xy^2$

Example 5: Factorize (x3 + 8y3 + 27z3 – 18xyz) using standard algebraic identities.

Solution: $(x^3 + 8y^3 + 27z^3 – 18xyz)$ is of the form Identity VIII where a = x, b = 2y and c = 3z. So we have,

$(x^3 + 8y^3 + 27z^3 – 18xyz) = (x)^3 + (2y)^3 + (3z)^3 – 3(x) (2y) (3z) = (x + 2y + 3z) (x^2 + 4y^2 + 9z^2 – 2xy – 6yz – 3zx)$

Applying Identities

The above identities are useful in carrying out squares and products of algebraic expressions. They provide us with easy alternative methods to calculate products of numbers and so on.

Addition and Subtraction of Algebraic Expressions

When we are adding or subtracting two algebraic expressions, we can only add or subtract like terms. The sum of two or more like terms is a like term, with a numerical coefficient equal to the sum of the numerical coefficient of all the like terms.

Similarly, the difference between two like terms is a like term with a numerical coefficient equal to the difference between the numerical coefficients of the two like terms.

Suppose if we have to add $3x^2y + y + z$ and $4x^2y + 7a + 5z$, we will combine all the like terms and then add their numerical coefficients.

$(3x^2y + 4x^2y) + (y) + (7a) + (z + 5z) = 7x^2y + y + 7a + 6z$

Multiplication of Algebraic Expressions

In Maths, Algebra is one of the important branches. The concept of algebra is used to find the unknown variables or unknown quantity. The multiplication of algebraic expressions is a method of multiplying two given expressions consisting of variables and constants. Algebraic expression is an expression that is built by the combination of integer constants and variables. For example, 4xy + 9, in this expression x and y are variables whereas 4 and 9 are constants. The value of an algebraic expression changes according to the value chosen for the variables of the expressions.

If there are brackets given in any expression, then it should be simplified first. When there is no bracket present, then the algebraic expressions can also be solved by applying division and multiplication, and then addition and subtraction, similar to BODMAS rule.

For a clear idea on this let us take an expression $2x+1$. Now if $x = 1$, the value of expression would be 3. If $x = 2$ the value will be 5 and so on. The value of the expression is dependent on the value of the variable. And if we have $2(x+1)$, then we will not get the same answer as we have got earlier. Here, if we put $x = 1$, then it gives $2(1+1) = 4$. So, we can see the difference when we use brackets in expressions.

Multiplication of Monomials

When we multiply two monomials:

the numerical coefficient of the terms is equal to the product of the numerical coefficient of both the terms.

the exponent or power of each algebraic factor is equal to the sum of the exponents of that algebraic factor in both the monomials.

Multiplying two monomials:

$X \times 3y = x \times 3 \times y = 3 \times x \times y = 3xy$

$3x \times 2y = 3 \times x \times 2 \times y = 3 \times 2 \times x \times y = 6xy$

$5x \times (-2z) = 5 \times (-2) \times x \times z = -10xz$

Multiplying three or more monomials:

$2x \times 3y \times 5z = (2x \times 3y) \times 5z = 6xy \times 5z = 30xyz$

$4xy \times 5x^2y^2 \times 6x^3y^3 = (4xy \times 5x^2y^2) \times 6x^3y^3 = 20x^3y^3 \times 6x^3y^3 = 120x^6y^6$

Distributive Property of Multiplication

The distributive property is an algebraic property that is used to multiply a single value and two or more values within a set of parenthesis.

Consider the expression: $6 \times (2 + 4x)$

$= (6 \times 2) + (6 \times 4x)$

$= 12 + 24x$

Here, we have used distributive law to multiply a monomial and a binomial.

Multiplying Two Monomials

In the product of two monomials

Coefficient = coefficient of the first monomial × coefficient of the second monomial

Algebraic factor = algebraic factor of a first monomial × algebraic factor of the second monomial

Multiplying Three or More Monomials

We first multiply the first two monomials and then multiply the resulting monomial by the third monomial. This method can be extended to the product of any number of monomials.

Rules of Signs

The product of two factors is positive or negative accordingly as the two factors have like signs or unlike signs. Note that

(i) (+) × (+) = +

(ii) (+) × (-) = –

(iii) (-) × (+) = –

(iv) (-) × (-) = +

If x is a variable and p, q are positive integers, then $x^p \times x^q = x^{p+q}$

Multiplication of any Polynomial

When we multiply any two polynomials, we multiply all the terms or monomials of one polynomial with all the terms of another polynomial.

When we multiply two binomials, every term in one binomial multiplies every term in the other binomial.

Multiplying a binomial by a binomial

(3a + 4b) × (2a + 3b)

= 3a × (2a + 3b) + 4b × (2a + 3b)

= (3a × 2a) + (3a × 3b) + (4b × 2a) + (4b × 3b)

= $6a2 + 9ab + 8ab + 12b^2$

= $6a2 + 17ab + 12b^2$

When we multiply a binomial by a trinomial, each of the three terms of the trinomial is multiplied by each of the two terms of the binomial.

Multiplying a binomial by a trinomial

$(p + 4) \times (p^2 + 2p + 3)$

$= p \times (p^2 + 2p + 3) + 4 \times (p^2 + 2p + 3)$

$= (p^3 + 2p^2 + 3p) + (4p^2 + 8p + 12)$

$= p^3 + 6p^2 + 11p + 12$

Horizontal method of multiplication

We are able to carry out the multiplication term by term using the distributive law.

Consider two binomials, say, (x + y) and (u + v).

Thus, we have, (x + y) × (u + v) = x × (u + v) + y × (u + v).

That is, $(x + y) \times (u + v) = (x \times u + x \times v) + (y \times u + y \times v)$

That is, $(x + y) \times (u + v) = xu + xv + yu + yv$

This method is also known as horizontal method for multiplication.

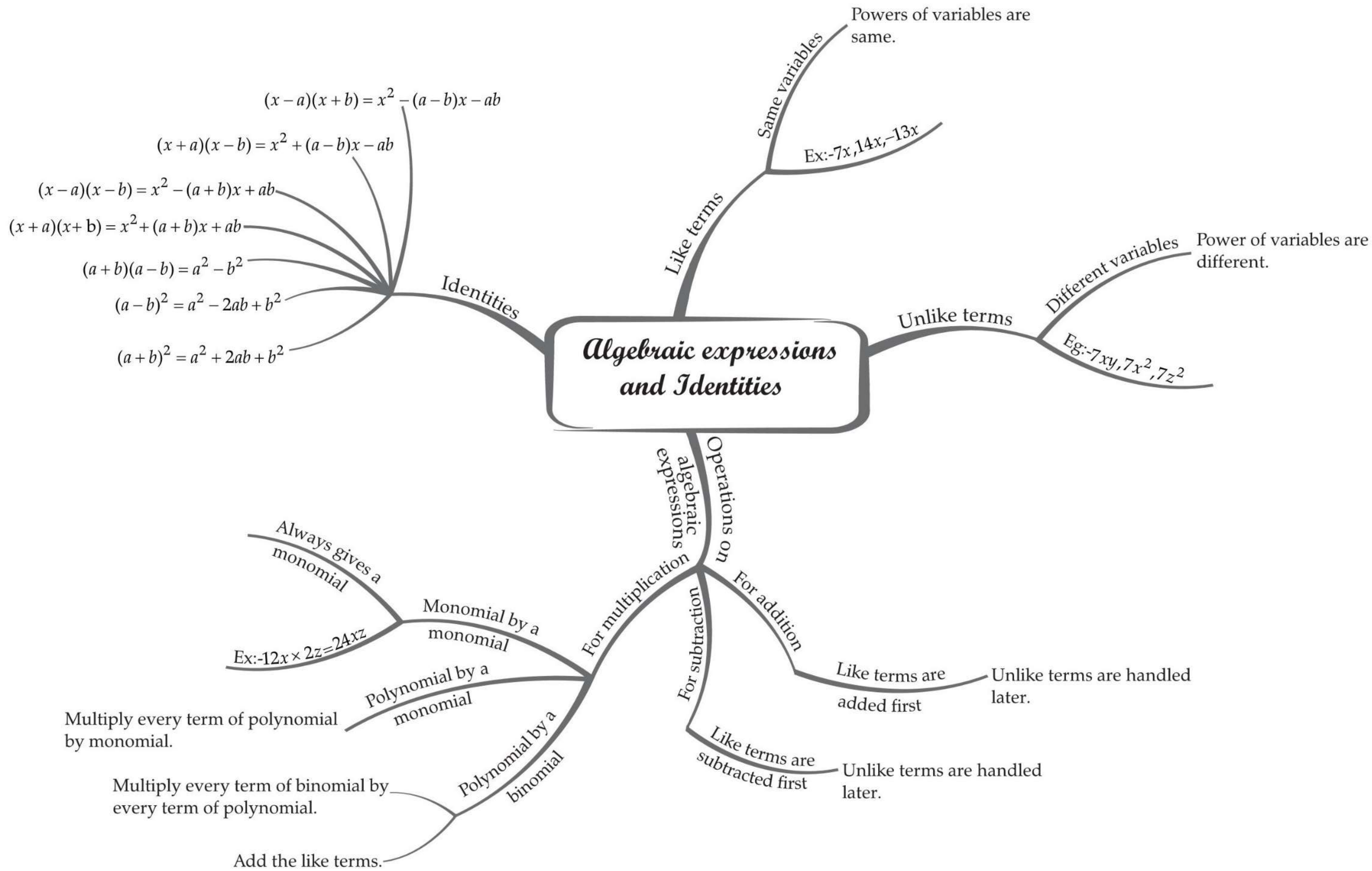
Column method of multiplication

Column method of multiplication of polynomials is very similar to the multiplication of two whole numbers. This method is used when the binomials being multiplied contain terms with like terms.

Example:

$$
\begin{array}{rl}
4x + 3y & \\
\times \quad 5x + 2y & \\
\hline
20x^2 + 15xy \quad\quad & \text{multiplying 4x+3y by 5x} \\
+8xy + 6y^2 & \text{multipllying 4x+3y by 2y} \\
\hline
20x^2 + 23xy + 6y^2 & \text{adding the like terms} \\
\hline
\end{array}
$$

MIND MAP : LEARNING MADE SIMPLE CHAPTER-9



Important Questions

Multiple Choice Questions-

Question 1. The expression x + 3 is in

(a) one variable

(b) two variables

(c) no variable

(d) none of these.

Question 2. The expression 4xy + 7 is in

(a) one variable

(b) two variables

(c) no variable

(d) none of these.

Question 3. The expression x + y + z is in

(a) one variable

(b) no variable

(c) three variables

(d) two variables.

Question 4. The value of 5x when x = 5 is

(a) 5

(b) 10

(c) 25

(d) -5.

Question 5. The value of $x^2 - 2x + 1$ when x = 1 is

(a) 1

(b) 2

(c) -2

(d) 0.

Question 6. The value of $x^2 + y^2$ when x = 1, y = 2 is

(a) 1

(b)2

(c) 4

(d) 5.

Question 7. The value of $x^2 - 2yx + y^2$ when $x = 1$, $y = 2$ is

(a) 1

(b) -1

(c) 2

(d) -2.

Question 8. The value of $x^2 - xy + y^2$ when $x = 0$, $y = 1$ is

(a) 0

(b) -1

(c) 1

(d) none of these.

Question 9. Which of the following is a monomial ?

(a) $4x^2$

(b) $a + 6$

(c) $a + 6 + c$

(d) $a + b + c + d$.

Question 10. Which of the following is a binomial?

(a) $3xy$

(b) $4l + 5m$

(c) $2x + 3y - 5$

(d) $4a - 7ab + 3b + 12$.

Very Short Questions:

1. Write two examples of each of

(i) Monomials

(ii) Binomials

(iii) Trinomials

2. Identify the like expressions.

$5x$, $-14x$, $3x2 + 1$, x^2, $-9x^2$, xy, $-3xy$

3. Identify the terms and their coefficients for each of the following expressions:

(i) $3x^2y - 5x$

(ii) $xyz - 2y$

(iii) $-x - x^2$

4. Add: $-3a^2b^2$, $\frac{-5}{2}a^2b^2$, $4a^2b^2$, $\frac{2}{3}a^2b^2$

5. Add: $8x^2 + 7xy - 6y^2$, $4x^2 - 3xy + 2y^2$ and $-4x^2 + xy - y^2$

6. Subtract: $(4x + 5)$ from $(-3x + 7)$

7. Subtract: $3x^2 - 5x + 7$ from $5x^2 - 7x + 9$

8. Multiply the following expressions:

(a) $3xy^2 \times (-5x^2y)$

(b) $\frac{1}{2}x^2yz \times \frac{2}{3}xy^2z \times 15\,x^2yz$

Short Questions:

1. Find the area of the rectangle whose length and breadths are $3x^2y$ m and $5xy^2$ m respectively.

2. Multiply $x^2 + 7x - 8$ by $-2y$.

3. Simplify the following:

(i) $a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2)$

(ii) $x^2(x - 3y^2) - xy(y^2 - 2xy) - x(y^3 - 5x^2)$

4. Multiply $(3x^2 + 5y^2)$ by $(5x^2 - 3y^2)$

5. Multiply $(6x^2 - 5x + 3)$ by $(3x^2 + 7x - 3)$

6. Simplify:

$2x^2(x + 2) - 3x(x^2 - 3) - 5x(x + 5)$

7. Multiply $x^2 + 2y$ by $x^3 - 2xy + y^3$ and find the value of the product for $x = 1$ and $y = -1$.

8. what must be added to $y^2 + 5y - 6$ to get $y^3 - y^2 + 3y - 2$?

9. What must be subtracted from $x^3 - 3x^2 + 5x - 1$ to get $2x^3 + x^2 - 4x + 2$?.

Long Questions:

1. Using suitable identity find:

(i) 482

(ii) 962

(iii) 2312 – 1312

(iv) 97 × 103

(v) 1812 – 192 = 162 × 200

2.

If $x^2 + \frac{1}{x^2} = 38$, find the values of:

(i) $x - \frac{1}{x}$ (ii) $x^4 + \frac{1}{x^4}$

3. Verify that $(11pq + 4q)^2 - (11pq - 4q)^2 = 176pq^2$

4. Find the value of $\frac{38^2 - 22^2}{16}$, using a suitable identity.

5. Find the value of x, if $10000x = (9982)^2 - (18)^2$

Answer Key-

Multiple Choice questions-

1. (a) one variable
2. (b) two variables
3. (c) three variables
4. (c) 25
5. (d) 0
6. (d) 5
7. (a) 1
8. (c) 1
9. (a) $4x^2$
10. (b) $4l + 5m$

Very Short Answer:

1. (i) Monomials:

   (a) $3x$

   (b) $5xy^2$

   (ii) Binomials:

   (a) $p + q$

   (b) $-5a + 2b$

   (iii) Trinomials:

   (a) $a + b + c$

   (b) $x^2 + x + 2$

2. Like terms: $5x$ and $-14x$, x^2 and $-9x^2$, xy and $-3xy$

3.

	Terms	Coefficients
(i)	$3x^2y$	3
	$-5x$	-5
(ii)	xyz	1
	$-2y$	-2
(iii)	$-x$	-1
	$-x^2$	-1

4.

$$(-3a^2b^2)+\left(-\frac{5}{2}a^2b^2\right)+(4a^2b^2)+\left(\frac{2}{3}a^2b^2\right)$$
$$=\left(-3-\frac{5}{2}+4+\frac{2}{3}\right)a^2b^2$$
$$=\left(\frac{-18-15+24+4}{6}\right)a^2b^2$$
$$=\frac{5}{6}a^2b^2$$

5.

$$\begin{array}{rrrr} & 8x^2 & +\,7xy & -\,6y^2 \\ & 4x^2 & -\,3xy & +\,2y^2 \\ & -\,4x^2 & +\,xy & -\,y^2 \\ + & & & \\ \hline \text{Sum} & 8x^2 & +\,5xy & -\,5y^2 \\ \hline \end{array}$$

6. $(-3x + 7) – (4x + 5) = -3x + 7 – 4x – 5 = -3x – 4x + 7 – 5 = -7x + 2$

7. $(5x^2 – 7x + 9) – (3x^2 – 5x + 7)$

$= 5x^2 – 7x + 9 – 3x^2 + 5x – 7$

$= 5x^2 – 3x^2 + 5x – 7x + 9 – 7$

$= 2x^2 – 2x + 2$

8.

(a) $3xy^2 \times (-5x^2y)$

$= (3) \times (-5) \cdot x^3y^3$

$= -15x^3y^3$

(b) $\frac{1}{2}x^2yz \times \frac{2}{3}xy^2z \times \frac{1}{5}x^2yz$

$$=\left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{5}\right)\cdot x^2yz \times xy^2z \times x^2yz$$
$$=\frac{1}{15}x^5y^4z^3$$

Short Answer:

1. Length = $3x^2y$ m, breadth = $5xy^2$ m

Area of rectangle = Length × Breadth = $(3x^2y \times 5xy^2)$ sq m = $(3 \times 5) \times x^2y \times xy^2$ sq m = $15x^3y^3$ sq m

2.

$$\begin{array}{r} x^2 \;+\; 7x \;-8 \\ \times -2y \\ \hline -2x^2y - 14xy + 16y \\ \hline \end{array}$$

3. (i) $a^2 (b^2 – c^2) + b^2 (c^2 – a^2) + c^2 (a^2 – b^2)$

$= a^2b^2 – a^2c^2) + b^2c^2 – b^2a^2) + c^2a^2 – c^2b^2)$

$= 0$

(ii) $x^2(x – 3y^2) – xy(y^2 – 2xy) – x(y^3 – 5x^2)$

$= x^3 – 3x^2y^2 – xy^3 + 2x^2y^2 – xy^3 + 5x^3$

$= x^3 + 5x^3 – 3x^2y^2 + 2x^2y^2 – xy^3 – xy^3$

$= 6x^3 – x^2y^2 – 2xy^3$

4. $(3x^2 + 5y^2) \times (5x^2 – 3y^2)$

$= 3x^2(5x^2 – 3y^2) + 5y^2(5x^2 – 3y^2)$

$= 15x^4 – 9x^2y^2 + 25x2y^2 – 15y^4$

$= 15x^4 + 16x^2y^2 – 15y^4$

5. $(6x^2 – 5x + 3) \times (3x^2 + 7x – 3)$

$= 6x^2(3x^2 + 7x – 3) – 5x(3x^2 + 7x – 3) + 3(3x^2 + 7x – 3)$

$= 18x^4 + 42x^3 – 18x^2 – 15x^3 – 35x^2 + 15x + 9x^2 + 21x – 9$

$= 18x^4 + 42x^3 – 15x^3 – 18x^2 – 35x^2 + 9x^2 + 15x + 21x – 9$

$= 18x^4 + 27x^3 – 44x^2 + 36x – 9$

6. $2x^2(x + 2) – 3x(x^2 – 3) – 5x(x + 5)$

$= 2x^3 + 4x^2 – 3x^3 + 9x – 5x^2 – 25x$

$= 2x^3 – 3x^3 – 5x^2 + 4x^2 + 9x – 25x$

$= -x^3 – x^2 – 16x$

7. $(x^2 + 2y) \times (x^3 – 2xy + y3)$

$= x^2(x^3 – 2xy + y^3) + 2y(x^3 – 2xy + y^3)$

$= x^5 – 2x^3y + x^2y^3 + 2x^3y – 4xy^2 + 2y^4$

$= x^5 + x^2y^3 – 4xy^2 + 2y^4$

Put x = 1 and y = -1

$= (1)^5 + (1)^2\,(-1)^3 - 4(1)(-1)^2 + 2(-1)^4$

$= 1 + (1)\,(-1) - 4(1)(1) + 2(1)$

$= 1 - 1 - 4 + 2$

$= -2$

8.

$$\begin{array}{r} y^3 - y^2 + 3y - 2 \\ y^2 + 5y - 6 \\ (-)\quad (-)\quad (+) \\ \hline y^3 - 2y^2 - 2y + 4 \\ \hline \end{array}$$

9.

$$\begin{array}{r} x^3 - 3x^2 + 5x - 1 \\ 2x^3 + x^2 - 4x + 2 \\ (-)\quad (-)\quad (+)\quad (-) \\ \hline -x^3 - 4x^2 + 9x - 3 \\ \hline \end{array}$$

Long Answer :

1.

(i) $(50-2)^2 = (50)^2 - 2 \times 50 \times 2 + (2)^2$

$= 2500 - 200 + 4$

$= 2504 - 200$

$= 2304$

(ii) $96^2 = (100-4)^2$

$= (100)^2 - 2(100)(4) + (4)^2$

$[\because (a-b)^2 = a^2 - 2ab + b^2]$

$= 10000 - 800 + 16$

$= 10016 - 800$

$= 9216$

(iii) $231^2 - 131^2$

$= (231 - 131)(231 + 131)$

$= (100)(362)$

$[\because a^2 - b^2 = (a + b)(a - b)]$

$= 36200$

(iv) 97×103

$= (100 - 3) \times (100 + 3)$

$= (100)^2 - (3)^2$

$[(a + b)(a - b) = a^2 - b^2]$

$= 10000 - 9$

$= 9991$

(v) $181^2 - 19^2 = (181 - 19)(181 + 19)$

$[\text{Using } a^2 - b^2 = (a - b)(a + b)]$

$= 162 \times 200 = 32400$

2.

(i) $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x}$

$[\because (a - b)^2 = a^2 + b^2 - 2ab]$

$= x^2 + \frac{1}{x^2} - 2$

$= 38 - 2 = 36$

$\therefore \quad x - \frac{1}{x} = \sqrt{36} = 6$

(ii) $\left(x^2 + \frac{1}{x^2}\right)^2$

$= x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2}$

$\Rightarrow \quad (38)^2 = x^4 + \frac{1}{x^4} + 2$

$\left[\text{Given} \left(x^2 + \frac{1}{x^2}\right) = 38\right]$

$\Rightarrow \quad 1444 - 2 = x^4 + \frac{1}{x^4}$

$\Rightarrow \quad 1442 = x^4 + \frac{1}{x^4}$

$\therefore \quad x^4 + \frac{1}{x^4} = 1442$

3. LHS = $(11pq + 4q)^2 - (11pq - 4q)^2 = (11pq + 4q + 11pq - 4q) \times (11pq + 4q - 11pq + 4q)$

[using $a^2 - b^2 = (a - b)(a + b)$, here $a = 11pq + 4q$ and $b = 11pq - 4q$]

$= (22pq)(8q)$

$= 176\ pq^2$

= RHS.

Hence Verified.

4.

Since $a^2 - b^2 = (a + b)(a - b)$, therefore

$$38^2 - 22^2 = (38 - 22)(38 + 22)$$
$$= 16 \times 60$$

So, $$\frac{38^2 - 22^2}{16} = \frac{16 \times 60}{16}$$
$$= 60$$

RHS = (9982)2 – (18)2 = (9982 + 18)(9982 – 18)

[Since a2 -b2 = (a + b) (a – b)]

= (10000) × (9964)

LHS = (10000) × x

Comparing L.H.S. and RHS, we get

10000x = 10000 × 9964

x = 9964